NUMBER SENSE

5 – 8

Number Sense
What All Students "Should Know"
5 – 8

1. Addition, subtraction, multiplication, and division with rational numbers.

Clarifications:

All students should:

- Know the meaning of rational numbers.

  Examples: ± 13; 0; $\sqrt{4}$; ± $\frac{13}{15}$ ± $\frac{7}{5}$ ±$3\frac{7}{10}$ ± $\frac{1}{3}$ ± $\frac{1}{9}$ ± .8 ; ± .83 ; ± 0.25

- Know the meaning of irrational numbers.

  Examples: $\sqrt{2}; \sqrt{3}; \Pi$ (pi)

- Recognize the reasonableness of answers when using the four operations of addition, subtraction, multiplication, and division with real numbers.

  Example:

$$\begin{array}{r} 21 \\ \times\ \underline{39} \end{array} \;\longrightarrow\; \begin{array}{r} 20 \\ \times\ \underline{40} \\ 800 \end{array}$$

  The answer should be close to 800.

- Utilize knowledge of magnitude (size) of rational numbers when using the four operations of addition, subtraction, multiplication, and division of rational numbers.
- Know the smallest number does not always divide largest number.

  Example: Answers may be a decimal or a fraction.

  $21 \div 70 = 0.3$ or $45 \div 60 = \frac{3}{4}$

- Know multiplication does not always produce a larger answer.

  Example: The answer is smaller than the original number when multiplying by a decimal or fraction less than one.

$$\begin{array}{r} 36 \\ \times\ \underline{0.35} \\ 12.60 \end{array}$$

- Know multiplication is repeated addition.

  Example: 13 X 5 = 13 + 13 + 13 + 13 + 13

2. Numbers and their relationships can be represented in multiple forms.

Clarifications:
All students should:

- Recognize equivalent forms of fractions.
  Examples:
    - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$
    - **.**5 = **.**50 = 0**.**50 = 0**.**5
    - $\frac{1}{2} =$ **.**5 = 50% = $ **.**50 = $\frac{4}{8}$
    - $2^3 = 8 = 2 * 2 * 2$
    - 3 to 4 = $\frac{3}{4}$ = 75%
    - $2300 = 2.3 \times 10^3$
    - $.056 = 5.6 \times 10^{-2}$
- Recognize symbols for relationships.
  Examples:
    - $<, >, \leq, \geq, =, \neq$
    - Proportion problems — solve by equivalent fractions or by cross products.
    - Distributive reasoning (distributive property) —

      $$\begin{array}{r} 23 \\ \underline{\times\ 15} \end{array}$$ is equivalent to (23 X 15) = (23 X 10) + (23 X 5) = 345 or

      (5 X 3) + (5 X 20) + (10 X 3) + (10 X 20)

3. The appropriate use of technology.

Clarifications:

All students should:

- Use technology as a tool to do problem solving beyond computation and checking answers.

  Examples:
  - ♦ Basic calculators
  - ♦ Scientific calculators
  - ♦ Graphing calculators
  - ♦ Computers
    - -- Internet
    - -- Spreadsheets

Teachers should:

- Use technology for teaching number sense.

  Examples:
  - ♦ Exploring negative numbers by starting at 99 and subtracting 5 as a constant.
  - ♦ Exploring exponents by starting at 3 and multiplying by 3 as a constant factor.
  - ♦ Exploring decimals by starting at 3**.**0 and adding **.**01 as a constant.
  - ♦ Exploring relationships between fractions and decimals by using the F ⇔ D key on a calculator with fraction functions.
  - ♦ Recursive ideas including now/next notation.

Number Sense
What All Students "Should Do"
5 – 8

Written Benchmark: A

Extend understanding and ability to apply whole number operations to all rational numbers (such as integers, fractions, and decimals).

<u>Problem 1</u>:

Process Standards: 1.10 and 3.3

At 5:00 p.m. the temperature was 21° F. Later that night the temperature dropped to 3°below zero. How many degrees did the temperature drop?

Solution Notes:
The temperature dropped 24°. The students can subtract 21 - (-3) or just count down from 21 to -3.

Prerequisites:
All students should:

1. Be able to subtract 21 - (-3) or just count down from 21 to -3.

<u>Problem 2</u>:

Process Standard: 3.2

Write a number sentence or expression that represents the number one (1) as the product or the sum of three different fractions. Explain the reasoning used on determining your fraction.

Solution Notes:
Example:

$$\frac{2}{7}+\frac{1}{7}+\frac{4}{7}=1$$

I chose these fractions because the fractions added up to $\frac{7}{7}$ = 1 and the numerators are different so they are different fractions.

$$\frac{10}{3}\times\frac{3}{7}\times\frac{7}{10}=1$$

I chose these fractions because they have different numerators and denominators and the numbers can simplify or can be multiplied to give $\frac{210}{210}=1$

$$\frac{\cancel{10}}{\cancel{3}} \quad X \quad \frac{\cancel{3}}{\cancel{7}} \quad X \quad \frac{\cancel{7}}{\cancel{10}} \quad = 1$$

Prerequisites:

All students should:

1. Know the process to determine common denominator and reciprocal.
2. Know how to add and subtract fractions.
3. Know how to multiply fractions.
4. Understand why the denominators change with multiplication and not with addition. (Connect combining like factors to adding fractions).

Number Sense
What All Students "Should Do"
5 – 8

Written Benchmark: B

Use multiple representations of equivalent forms of numbers such as integers, fractions, decimals, percent, exponents, and scientific notation in a variety of situations.

Problem 1:

Process Standards: 1.6 and 2.1

Choose a number between 1/5 and 1/4. Compose a paragraph in which you use the number between 1/5 and 1/4 that you chose. Include in your paragraph both the fraction and the decimal equivalent forms of your number.

Solution Notes:

1/5 = .20 and 1/4 = .25

The decimal solutions range from .201 to .249. Any fraction solutions range from $\frac{201}{1000}$ to $\frac{249}{1000}$

The context of the paragraph may vary.

Also:

$$\frac{1}{5} = \frac{4}{20} = \frac{8}{40}$$

$$\frac{9}{40} \text{ is between } \frac{1}{5} \text{ and } \frac{1}{4}$$

$$\frac{1}{4} = \frac{5}{20} = \frac{10}{40}$$

Prerequisites:

All students should:

1. Know how to change a decimal to a percent.
2. Know how to change a decimal to a fraction.
3. Know how to change a fraction to a percent.
4. Know how to change a percent to a fraction.
5. Know how to change a fraction to a decimal.
6. Know how to change a percent to a decimal.
7. Know how to change two or more fractions to equivalent fractions with common denominators.

Problem 2:
Process Standard: 1.8

Rank the following numbers in order from least to greatest: $.35; \frac{3}{5}; 42\%; 2.3\times10^{-2}$.

Determine which number is closest to the whole number 1. Provide the work or explanation that shows how you arrived at your answer.

Solution Notes:

Method 1:

$2.3 \text{ X } 10^{-2}$; .35 ; 42% ; $\frac{3}{5}$

$\frac{3}{5}$ is the number closest to the whole number 1.

Justifications may vary. If you convert all of the numbers to decimals and order them, you get .023 ; .35 ; .42 ; .60. $\frac{3}{5}$ = .60 is the number closest to 1.

Method 2:

.35 = 35%

$\frac{3}{5}$ = 60%

42% = 42%

$2.3 \text{ X } 10^{-2}$ = .023 = 2.3%

The fraction $\frac{3}{5}$ is closest to one (1) because I changed all the numbers to % and 60% was the closest to 100% (or 1), so 60% = $\frac{3}{5}$ and would be the closest to 1.

Prerequisites:

All students should:

1. Know how to change fractions to decimals.
2. Know how to change a fraction to a decimal to a percent.
3. Understand exponents and scientific notation.
4. Know the magnitude of decimals.

Problem 3:
Process Standards: 3.5 and 4.1

Activity:
Each student is asked to write a fraction, decimal, or percentage on an index card. Then all students are asked to go to the front of the room (WITHOUT TALKING!) with their number and line up from greatest to least. When they are called on, they must tell why their number goes between the two numbers of the two people they are standing between.

Teacher note: Have a plan for the instructions when students select the same or equivalent fractions.

Solution Notes and Evaluation:
Students must answer why the number they are holding is placed between the numbers the people are holding that are standing on either side of them. A student's explanations may refer to equivalent forms of their value and those held by the students on either side of them, for example: 1/5 belongs between 25% and .18 because the decimal for 25% is .25, the decimal for 1/5 is .20 and .25 > .20 > .18.

Prerequisites:
All students should:

1. Be able to change a decimal to a percent and to a fraction.
2. Be able to make comparisons of values using >, <, $\geq$, $\leq$.
3. Know the magnitude of fractions, decimals, and percents.

Number Sense
What All Students "Should Do"
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Written Benchmark: C

Describe connections and relationships of numbers such as ratios, proportions, and percents in problem situations.

<u>Problem 1</u>:
Process Standard: 3.1

Central Middle School has earned an ice cream party. Three out of four students prefer strawberry ice cream. One out of six students prefer chocolate ice cream. The rest of the students prefer vanilla. Based on this information, out of the total student population of 900, how many prefer vanilla? Express your answer as a ratio and give the total number of students who prefer vanilla ice cream. Explain your solution.

Solution Notes:
3/4 = 9/12 and three out of every 4 students like strawberry, and since one out of six like chocolate, then

$$\frac{9}{12}+\frac{1}{6}=\frac{9}{12}+\frac{2}{12}=\frac{11}{12}; \quad 1-\frac{11}{12}=\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$$

Since one is the total of all three ratios, by subtracting, the number of students preferring vanilla is $\frac{1}{12}$. $\frac{1}{12}$ X 900 students = 75 students preferring vanilla ice cream.

Prerequisites:
All students should:

1. Know how to find the common denominator for two or more fractions with unlike denominators.
2. Be able to work problems involving proportions.
3. Be able to add and subtract fractions.
4. Be able to multiply fractions.

<u>Problem 2</u>:
Process Standard: 1.3

Day 1:
Give the students meter sticks to take outside with them to use for the activity.
When the students get outside, they are asked to measure the lengths of each person's shadow and compare that length to each person's height (in same units) to look for patterns and relationships.

Day 2:
Students go outside and are asked to determine the height of the flagpole or a tree they cannot possibly measure. The height they are determining must be in meters. Suggest that the students consider the relationships and patterns from the previous day.

Solution Notes:
(These notes should be used to guide discussions not simply given without student involvement or development.)
Have the students stand the meter stick on its end $\perp$ (perpendicular) to the ground. Measure the shadow cast by the meter stick. Set up a proportion using the ratio:

$$\frac{\text{Length of shadow of meter stick}}{\text{1 meter}}$$

Measure the shadow of the flagpole or object of unknown height. Set up that ratio:

$$\frac{\text{Length of shadow of the object of unknown height}}{x},$$

where x = height of unknown object.

Set the ratios equal to make a proportion and solve for X.

$$\frac{\text{Length of shadow of meter stick}}{\text{1 meter}} = \frac{\text{Length of shadow of the object of unknown height}}{X}$$

Prerequisites:
All students should:
1. Be able to measure using metric measurement.
2. Be able to work problems involving proportions and determine the unknown quantity that is a part of the proportion.

Extension:
Have students (or another class) go out and repeat the procedure, but at a different time of the day. Have the students use the same objects to measure and determine if they get the same results. Have the students explain orally or write a paragraph explaining why the lengths of the shadows are different, but the height of the object remains the same.

Number Sense
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Written Benchmark: D

Investigate number forms such as fractions, decimals, and percents, and demonstrate their uses in today's society.

Problem 1:
Process Standard: 1.8

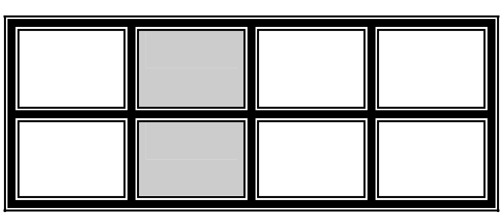
You plan to share a KING SIZE candy bar with three of your friends. Just as you are unwrapping the candy bar, your two cousins and their two friends show up. Then you must share the candy bar with everyone. Draw a picture illustrating how everyone can share this candy bar equally. Shade the part given to your cousins. What percent of the candy bar did they receive?

Solution Notes:
There are more than two ways to divide the candy into eight (8) equal parts. One way is to use a scaled ruler and measure the length then divide if into four equal parts and divide the width of the candy bar into half. A second way to divide the candy bar into eighths is to use the diagonals of each half. Draw diagonals for the rectangular candy bar. Divide the candy bar in half by drawing a $\perp$ line at the intersection of diagonals. Repeat the diagonal division for the halves until you get eight equal parts. Shade two of the parts to represent the parts given to the cousins.

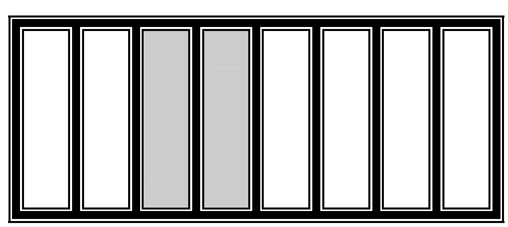
Find the percent: 2/8 = 1/4 = 25%

1.

2.



3.



Problem 2:
Process Standard: 1.10

Because of good class behavior, Mr. Michaelson was treating his class to donuts. He placed an order for three-dozen donuts, and later realized he had made a mistake. He placed a call to The Donut Shop and increased his order by 125%! What was the total number of donuts delivered to Mr. Michaelson's class?

Solution Notes:

Method 1:

$1 \text{ dozen} = \frac{12 \text{ donuts}}{\text{dozen}}$ $\quad$ $3 \text{ dozen} \times \frac{12 \text{ donuts}}{\text{dozen}} = 36 \text{ donuts}$

36 donuts X 125% = 36 donuts X 1.25 = 45 donuts

36 donuts + 45 donuts = 81 donuts

Method 2:

Step 1

$1 \text{ dozen} = \frac{12 \text{ donuts}}{\text{dozen}}$

$3 \text{ dozen} \times \frac{12 \text{ donuts}}{\text{dozen}}$

3 X 12 donuts = 36 donuts

Step 2

36 donuts + 36 donuts X 125%

$36 \text{ donuts} + \underbrace{36 \text{ donuts} \times 1.25}_{45 \text{ donuts}}$

36 donuts + 45 donuts

Total = 81 donuts

Method 3:

1 dozen = 12 donuts
3 dozen = 3 X 12 = 36 donuts
Push into my calculator:
36 + 125%
I get 81 total donuts.

Method 4:

100 % increase means it doubles; so

$$\begin{array}{r} 36 \\ +36 \\ \hline 72 \text{ donuts} \end{array}$$

$25\% = \frac{1}{4}$ and $\frac{1}{4}$ of 36 =

$$\begin{array}{r} 72 \\ +\ 9 \\ \hline 81 \text{ donuts} \end{array}$$

Mr. Michaelson's class would have 81 donuts coming to class! Yeah for his class!

Prerequisites:

All students should:

1. Understand percents > 100.
2. Understand the meaning of dozen.
3. Understand how to use a calculator to solve problems.

Number Sense
What All Students "Should Do"
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Written Benchmark: E

Develop, analyze, and explain procedures for computation and techniques for estimation.

<u>Problem 1</u>:

Process Standards: 3.5

Yesterday's math assignment involved division of decimals. The directions said to round the final answer to the nearest tenth. The answer to the last problem was rounded to 37.5. What was the range of numbers that would produce the answer 37.5?

Solution Notes:

Method 1:
In order to get the final answer of 37.5, the hundredths digit could have been 0, 1, 2, 3, 4 and that would leave the 5 tenths unchanged. The student could have started with 37.4 and had big numbers like 5, 6, 7, 8, and 9 after the 4 (in the hundreds place) and that would have caused the 37.4 to be rounded to 37.5. So the range of numbers that produced the answer of 37.5 would be 37.45 - 37.54.

Method 2:
37.5 was the answer so the numbers could be rounded by looking at the number to the right of the tenths place. <u>37.5</u> | that number could be:

<u>37.5</u>| 0	<u>37.5</u>| 5
<u>37.5</u>| 1	<u>37.5</u>| 6
<u>37.5</u>| 2	<u>37.5</u>| 7
<u>37.5</u>| 3	<u>37.5</u>| 8
<u>37.5</u>| 4	<u>37.5</u>| 9
These numbers will not round it up so it would stay 37.5 which is what should happen.	These numbers round the *5* up to a 6 so the answer would be 37.6*,* which does not work.

Most students would stop here! But they should go on if they have an understanding of rounding:

37.4 | 0	37.4 | 5
37.4 | 1	37.4 | 6
37.4 | 2	37.4 | 7
37.4 | 3	37.4 | 8
37.4 | 4	37.4 | 9

These numbers round the 4 up to a 5, which is what is needed to get 37.5. The range would be 37.54 - 37.45.

Problem 2:
Process Standards: 3.5

Using the situation in Problem 1, how should numbers between 37.540 and 37.54999… be rounded? Explain your answer.

Solution notes:
They should all be rounded to 37.5. Students should demonstrate an understanding and recognition of the denseness of decimal fractions (e.g., between any two decimal numbers there exists another decimal number).

Prerequisites:
All students should:
1. Understand the rounding of decimal numbers.
2. Know what is meant by range.
3. Know the decimal place values.

Problem 3:
Process Standards: 3.5

What effect does dividing a whole number by a fraction have on its size? Provide at least four examples (one of which must be an improper fraction) and then draw a conclusion based on information from your examples. Write your conclusion in sentence form.

Solution Notes:
Students may not realize that dividing a whole number by a fraction between 0 and 1 increases the magnitude (size) of the number. The students should be

encouraged to provide some examples (including improper fractions—to test conclusions) of dividing a whole number by a fraction:

$2 \div \frac{1}{2}$ = 4; $3 \div \frac{1}{4}$ = 12; $4 \div \frac{2}{3}$ = 6; $10 \div \frac{5}{4}$ = 8

Examples may vary. Conclusions may vary.

Conclusion: Let's take 4 ÷ ½ as an example. 4 ÷ ½ means you have four items or groups. You want to take each one of the four groups and divide each in ½, therefore, ending up with eight items or groups. However, dividing by a fraction larger than one makes the number get smaller.

Prerequisites:

All students should:

1. Know how to divide fractions.

Number Sense
What All Students "Should Do"
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Written Benchmark: F

Develop, analyze, and explain methods for solving proportions.

Problem 1:
Process Standards: 1.6 and 3.6

John and Mike went to the arcade to play the Space Invasion Video Game. Mike played first and won three out of seven of his games. John wants to have a better record than Mike. If he plays nine games, what is the least number of games that he must win in order to have a better record?

Solution Notes:
The solution notes contain four methods of working this problem. More methods may be possible.

Method 1: Set up a proportion.

$$\frac{3}{7} = \frac{X}{9}$$

$7X = 27$

$X = 3\frac{6}{7}$

Since you cannot have 3 6/7 games, you increase it to four games that John must win to have a better record than Mike.

Method 2:
Mike won three out of seven or 42.8 % of his games.

$$\begin{array}{r|l} & .428 \\ \hline 7 & 3.00 \\ & \underline{2\ 8} \\ & 20 \\ & \underline{14} \\ & 60 \\ & \underline{56} \\ & 4 \end{array}$$

John played nine games to be better than Mike; John would have to win more than 42.8% of his games.

$$\begin{array}{r} .43 \\ \times 9 \\ \hline 3.87 \end{array} \qquad \begin{array}{r} .44 \\ \times 9 \\ \hline 3.96 \end{array} \qquad \begin{array}{r} .45 \\ \times 9 \\ \hline 4.05 \end{array}$$

The least number of games that John could win would be four games.

<u>Problem 2</u>:

Process Standard: 1.10

The Home Economics class is making cookies. The recipe calls for ¾ cup of sugar for thirty cookies. How much sugar is needed to make forty cookies?

Solution Notes:

$$\frac{\frac{3}{4}\text{ cup}}{30\text{ cookies}} = \frac{x\text{ cups}}{40\text{ cookies}}$$

So, 30 x = ¾ 40; then 30 x = 30; therefore x = 1 cup of sugar

Prerequisites:

All students should know how to use:

1. Ratios.
2. Proportions.
3. Understand the meaning of multiply.

Number Sense
What All Students "Should Do"
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Written Benchmark: G
Check and explain the reasonableness of solutions, strategies and results.

<u>Problem 1</u>:
Process Standard: 3.3

La Shae has wanted a new hat. She has found a hat that she likes. It costs $24.00. Her mom says that is too much! On Tuesday the store has a 40% off sale. Mom says it's still too expensive. Two weeks later another 20% is marked off the sale price. She tells her Mom "Now we can buy the hat because we'll be saving 60%." Her Mom doesn't believe that's right. Who understands percent, La Shae or her Mom? Justify your answer.

Solution Notes:
Method 1 :
La Shae's Mom has a better understanding of percentages. La Shae thinks that 40% off the original price and 20% off the sale price gives you 60% off the original price. This is not correct reasoning. The actual discount is as follows.

$24.00 - .40($24.00) - .20[$24.00 - .40($24.00)] = $11.52 is the sale price with both discounts.

$12.48 of $24.00 is a combined discount of 52% off of the original price.

Method 2:
I entered $24.00 - 40% into my calculator and the answer was 14.40; then I entered 14.40 - 20% and the answer was $11.52.
La Shae's method 24.00 - 60% is $9.60. These two prices are not the same, so La Shae's reasoning is wrong, because the 20% was off the sale price or subtracted from the first sale price of the hat, not the original price of the hat. [Note: Teacher needs to verify that student calculators have a percent key and include this function.]

Prerequisites:
All students should:
1. Know how to calculate percents.
2. Understand the meaning of the term discount.
3. Be calculator literate.

Problem 2:

Process Standard: 3.6

Jill and Sam were working on their math homework. Sam wrote 7.69 > 7.7. Jill said that statement was false. Sam explained that it had to be right because 7.69 is greater than 7.7. Which student's thoughts are correct? Why was their thinking correct? Finally, explain why the first student's explanation will not justify the differences in the sizes.

Solution Notes:

Method 1:

Jill was correct in telling Sam that 7.69 > 7.7 is a false statement. The difference between 7.69 and 7.7 is 0.01. This is a significant amount. Therefore you cannot conclude that 7.69 > 7.7. It seems as if Sam multiplied 7.69 by 100 to get 769, but he multiplied 7.7 by 10 to get 77, further distorting the applied logic. Whether Sam multiplies both sides by 10 or 100 to get 76.9 > 77.0 or 769 > 770, both statements are still false.

Method 2:

Jill was correct in saying the statement was false. Since both of the numbers have a 7 in the ones place, they are equal in value so far. Then you compare the next decimal place and notice that 6 is less than 7 so Sam is wrong. You don't have to look at any more decimal places because you have already found the smaller decimal (7.69).

Method 3:

Jill was correct because when you compare decimals it is sometimes easier if you have the same number of digits after the decimal point. So you add zeros. 7.69 > 7.7. We need to add a zero on the right side of the inequality, then 7.69 > 7.70. It is easy to see Jill was correct because if both sides are multiplied by 100, you get 769 > 770, which is a false statement.

Prerequisites:

All students should:

1. Know place values for whole numbers and decimals.
2. Know the meaning of inequality symbols [<, >, ≤, ≥].

Number Sense
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Written Benchmark: H

Select appropriate methods of computation, such as mental arithmetic, estimation, calculator, computer, and paper/pencil to reflect upon solutions, strategies, and results.

<u>Problem 1</u>:
Process Standard: 1.4

Create a spreadsheet to show the following entries and the closing balance.

March 1	Deposit paycheck	$452.50
March 3	Write a check for groceries	$ 72.45
March 4	Write a check for telephone bill	$ 46.30
March 8	Deposit paycheck	$452.50
March 9	Write a check for gas / electric bill	$127.88

Solution Notes:

Checking Account Entries and the Closing Balance				
Date	Description of Transaction	Withdrawal	Deposit	Balance
March 1	Deposit Pay Check		452.50	452.50
March 3	Write a check for groceries	$ 72.45		380.05
March 4	Write a check for Telephone	$ 46.30		333.75
March 8	Deposit Pay Check		452.50	786.25
March 9	Write check for the gas/electric bill	$127.88		658.37

Prerequisite Skills:
All students should:
1. Know how to use a spreadsheet program.
2. Know the vocabulary of deposits and withdrawals.

<u>Problem 2</u>:
Process Standards: 1.10 and 3.5

Samantha and her friend went to lunch at a restaurant. Their bill was **$**38.15. Samantha wants to leave a 15% tip. She does not have a calculator or pencil and paper, so she is trying to figure this in her head. Write out a procedure and describe in words a way she might figure the tip of 15% in her head. (Remember tips are approximate answers).

Solution Notes:
Samantha rounds $38.15 to $40 and takes 10% of 40 for $4. If she knows that 5% is half of 10%, then half of $4 is $2. So she owes 10% + 5% or $4 +$2 or $6.00 for the tip.

Prerequisites:
All students should understand:
1. The meaning of percents.

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Written Benchmark: I
Represent numerical relationships in one- and two-dimensional graphs.

<u>Problem 1</u>:
Process Standards: 1.8

Students were asked to rate a recent movie. Seventeen out of thirty students who saw the movie gave it a four star rating. On the graph, illustrate by shading what seventeen out of thirty would look like on the graph.

0 students 30 students

Sam said that this rating was greater than a 50% approval rating. Is Sam correct? Why was he correct or why was he not correct in his statement?

Solution Notes:

0 students 30 students

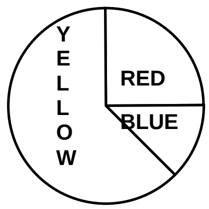
Yes Sam is correct because 50% is ½ so 50% of 30 is ½ of 30 or 15. Seventeen is greater than 15. [17 > 15]. Therefore 17 is greater than 50%.

Prerequisites:
All students should:
1. Know the magnitude of ratios.
2. Know the magnitude of percents.

<u>Problem 2</u>:
Process Standard: 1.8

Using a protractor, construct a circle graph that shows one out of four people prefer the color red; one out of eight people prefer the color blue. The remaining section of the circle graph should be labeled yellow. State the ratio of people who prefer the color yellow.

Solution Notes:



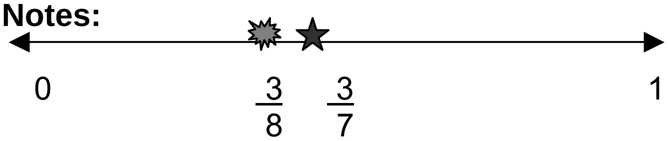
5 out of 8 people prefer yellow.

Prerequisites:
All students should know:
1. Fractional parts of a circle.
2. How to use a protractor.

<u>Problem 3</u>:
Process Standard: 3.5

The fraction 3/7 has been placed on the number line at its approximate location. If the denominator of the fraction is increased by one, a new fraction is created. Plot on the number line a point to represent the location of the new fraction. Explain in words the relationship between the original fraction and the new fraction where its denominator increased by one.

Solution Notes:
A.



B. If you start with a fraction 3/7 which is approximately equal to .429 and add one (1) to the denominator the new fraction is 3/8 and 3/8 is equal to .375, 3/8 < 3/7. Therefore by increasing the denominator by 1 your will decrease the magnitude (or size) of the fraction.

Prerequisites:
All students should know:
1. The vocabulary of fractions.
2. The magnitude of fractions.
3. How to add and subtract fractions.

GEOMETRIC AND SPACIAL SENSE

5 – 8

Geometric and Spatial Sense
What All Students "Should Know"
5 – 8

1. Structures of measurement systems.

Clarifications:

All students should know how to:

- Use the English or customary system of measurement:
  - ♦ Distance: inch, foot, yard, mil
  - ♦ Capacity: quart, gallon
  - ♦ Mass/weight: ounce, pound, ton
- Use the Metric system of measurement:
  - ♦ Distance: centimeter, meter, kilometer
  - ♦ Capacity: liter, kiloliter
  - ♦ Mass: gram, kilogram
- Use linear, square and cubic measurement and conversions within dimensional systems:
  - ♦ Square feet →square yards
  - ♦ Cubic feet → cubic yards
- Convert between English (customary) and Metric systems — [Know where to find the information and how to use the information to convert from one system to the other].
- Use measurement of time.
- Measure and classify angles:
  - ♦ Acute angle
  - ♦ Obtuse angle
  - ♦ Right angle
  - ♦ Straight angle
  - ♦ Adjacent angles
  - ♦ Supplementary angles
  - ♦ Complementary angles
  - ♦ Vertical angles
- Use ratios in scale drawings and the conversion process.

2. Description of two- and three-dimensional shapes and their relationships.

Clarifications:
All students should know:

- Basic descriptions of two- and three- dimensional shapes (figures):
  - ♦ Polygons (two-dimensional) – triangle, square, rectangle, pentagon....
  - ♦ Space figures (three-dimensional) – rectangular prism, rectangular pyramid, cylinder, sphere, cone, etc.
- Names of two- and three- dimensional shapes (figures) based upon the number of edges, faces and vertices.
- Classifications of polygons and polyhedral (three-dimensional space figures) based on the size and location of the angles – regular, concave, convex, etc.
- The meaning of circle and the terms related to circles – radius, diameter, arc, chord, etc.
- The classifications of triangles:
  - ♦ Scalene
  - ♦ Isosceles
  - ♦ Equilateral
  - ♦ Acute
  - ♦ Obtuse
  - ♦ Right
- Change in perimeter in relationship to change in area for all two-dimensional figures.
- The relationships of two-dimensional shapes to two-dimensional shapes:
  - ♦ Similar
  - ♦ Congruent
- The relationships of three-dimensional shapes to three-dimensional shapes:
  Example: Change in the volume for:
  - ♦ A cone is ⅓ of the volume of a cylinder if the bases remain constant.
  - ♦ Two rectangular prisms with congruent bases, the rectangular prism with the greatest height has the greatest volume.
  - ♦ Two cylinders with the same height, the cylinder with the greatest circumference has the greatest volume.
- The relationship of two-dimensional shapes to three-dimensional shapes:
  - ♦ Two-dimensional shapes have length and width with no depth while three-dimensional shapes have length, width, and depth.
  - ♦ The surface area of a three-dimensional shape is the sum of the areas of all the two-dimensional shapes covering the surface of the three-dimensional shape.

3. Geometric shapes are found in the real world.

Clarifications:

All students should know real-world applications in:

- Architecture and construction– trusses, tiling, windows, cabinets, golden ratio, symmetry, shape recognition, etc.
- Interior decorating – size and shape of pictures and furniture, placement of pictures and furniture, purchasing of carpet, etc.
- Nature – snowflakes, crystal formations, golden ratio, symmetry, and shape recognition.
- Consumer objects – cereal boxes, soda cans, etc.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: A

Identify, describe, compare, classify, and represent geometric figures.

<u>Problem 1</u>:

Process Standard: 3.3

Explain why you cannot have an obtuse equilateral triangle. Include sketches of triangles to support your explanation.

Solution Notes:

In an equilateral triangle all 3 angles are the same measure of 60°. To have an obtuse triangle, one angle must be greater than 90°.

<u>Method 1</u>:

This is a contradiction since a triangle only has three angles. If all three angles are 60° then one angle could not be greater than 90°. Therefore, it is not possible to have an obtuse equilateral triangle.

<u>Method 2</u>:

This is a contradiction since a triangle contains a total of 180° in the sum of all three angles. If a triangle has 3 angles and two of the angles are 60° and the third angle is greater than 90°, the sum of the angles is greater than 180°. Therefore, it is not possible to have an obtuse equilateral triangle.

Other possible conclusions may be correct, if the reasoning is based on correct information and the thought process is logical.

Prerequisites:

All students should:

1. Understand the different types of triangles.
2. Understand the different number of degrees in each angle of different types of triangles, and the total number of degrees in a triangle.
3. Length of sides related to opposite angles.

Problem 2:
Process Standards: 1.5, 2.1, 2.3, 3.2, and 3.3

Draw a card out of a pile of cards. Without revealing the term on the card, describe the figure to your partner as your partner draws the figure you are describing.

Solution Notes:
Place the following terms (some or all) on individual index cards.

Basic Geometric Terms

- Point
- Line
- Ray
- Prism
- Segment
- Pyramid
- Plane
- Parallel lines
- Perpendicular lines
- Intersecting lines
- Horizontal
- Vertical
- Diagonal/oblique

Angles

- Straight
- Right
- Acute
- Obtuse

Quadrilaterals

- Rectangle
- Square
- Rhombus
- Parallelogram
- Trapezoid

Triangles

- Acute
- Obtuse
- Right
- Isosceles
- Scalene
- Equilateral

Angles of Polygons and/or space Figures

- Pentagon
- Hexagon
- Rectangular
- Triangular
- Cone
- Sphere

Have students work in pairs. Have one student in the pair select a card and describe the attributes of the figure as the other student draws the figure based on the description. The name of the figure cannot be revealed until the other student in the pair has completed the drawing of the figure. Have the students check the drawing of the figures on the card to see if the student giving descriptions understands the concept. As an evaluation the teacher will go from pair to pair observing each student both giving descriptions and drawing the figures from the other student's descriptions.

Prerequisites:
All students should:

1. Know the geometric terms (listed above).

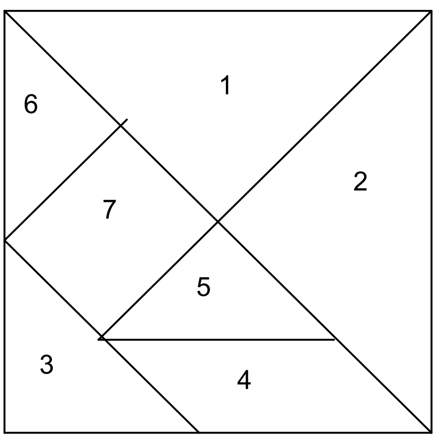
Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

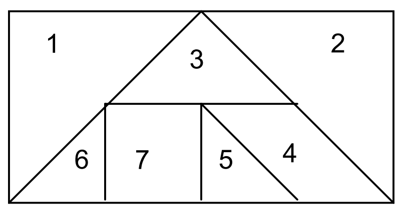
Written Benchmark: B
Explore transformations of geometric figures.

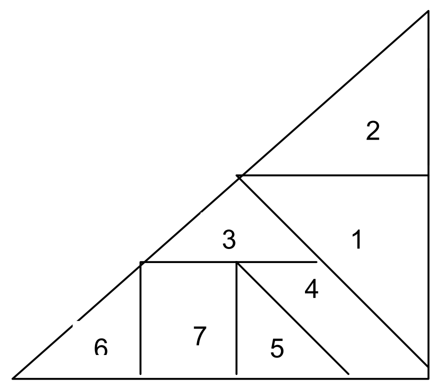
<u>Problem 1</u>:
Process Standards: 1.6 and 4.1

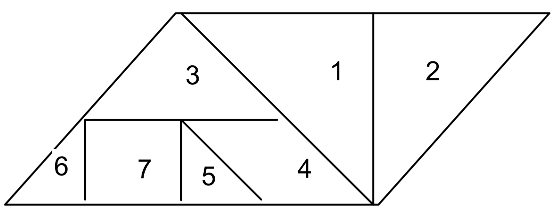
Using all seven pieces of a tangram set, create a square, rectangle, triangle, parallelogram, and a trapezoid.

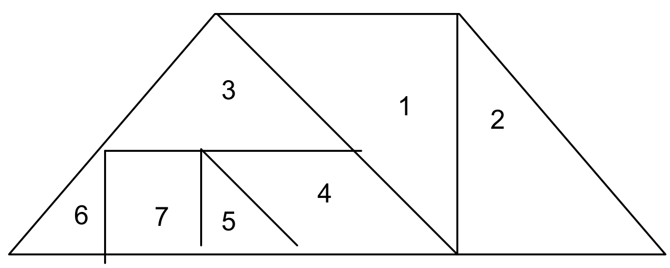
Solution Notes:
Students will see the relationships between shapes as they move from one shape to another.











Prerequisites:
All students should:

1. Know the attributes of a square, triangle, rectangle, parallelogram and trapezoid.

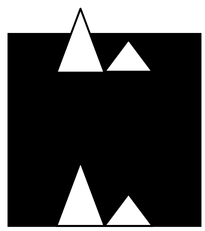
<u>Problem 2</u>:
Process Standards: 1.5, 1.6, and 2.1

Tessellation Project (Activity):
Begin with a 3-inch X 3 inch square (OR a basic shape that will tessellate). Make two changes to the square (by cutting out a piece of one side and translating or rotating it to another side) so that the new shape will tessellate (cover a sheet with no gaps). Make the shape, describe the process of changing to the new shape, and show the final new shape. Finally, trace the new shape and show the tessellation covering a 15-inch X 15 inch square.

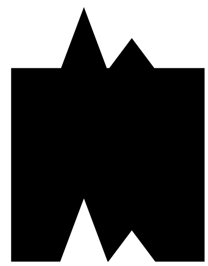
Extension:
Use technology/computer to design a tessellation.

Solution Notes:

Beginning Shape

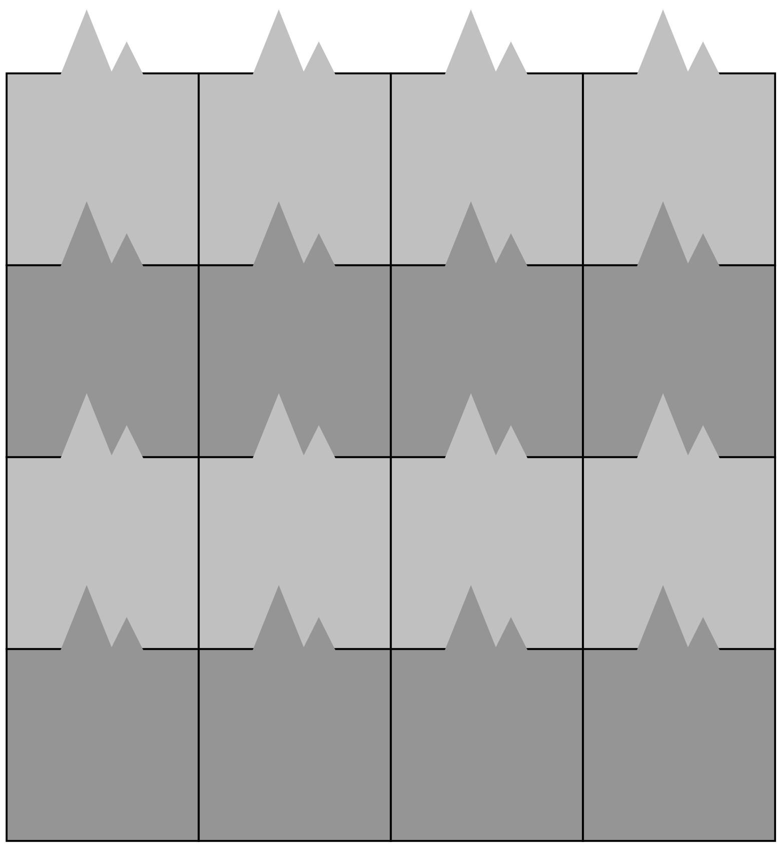


Changes



Final Shape

Will it tessellate? Yes



Prerequisites:
All students should:

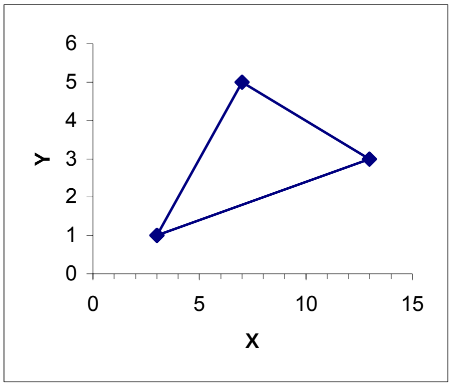
1. Have an understanding of tessellations.
2. Have had the process already modeled for them.

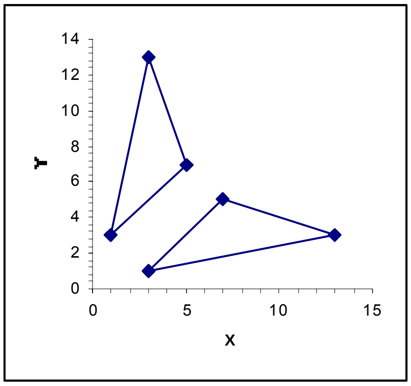
<u>Problem 3</u>:

Process Standards: 1.5, 1.6, and 2.1

In a coordinate plane, plot the points (7, 5), (3, 1), (13, 3) making a triangle. Then reflect those points across the *y* = *x* line. Identify the coordinates for the new triangle.

Solution notes:





The coordinates for the new triangle are (5,7), (1, 3), (3, 13).

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: C
Investigate and apply geometric properties and relationships.

<u>Problem 1</u>:
Process Standards: 1.10 and 3.5

How many tiles would it take to tile your classroom if each tile is one square foot? Sketch a diagram of your classroom and label the measurements. Show your work! If one carton of one square foot tiles contains twelve tiles, how many cartons must be purchased?

Solution Notes:
Answers will vary per classroom floor plan.

Prerequisites:
All students should:

1. Be able to find the area of a rectangular or an irregular shape.
2. Be able to measure in feet.

<u>Problem 2</u>:
Process Standards: 1.5, 1.6, 3.3, and 3.5

Compare and contrast the following figures:

Write five statements about how the two figures are similar, and five statements about how the two figures are different.

Solution Notes:
Answers will vary– Sample answers.

A diagram (Venn or student generated) could be used to illustrate the similarities and differences of the two figures.

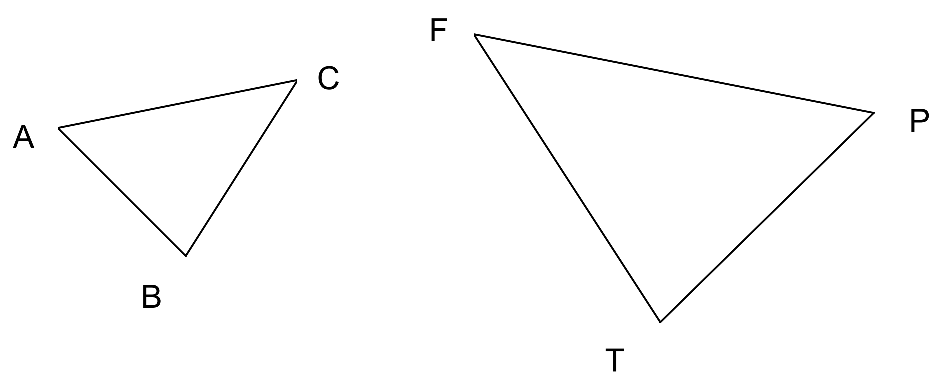
Similarities	Differences
Polygon	Lengths of bases
Quadrilaterals	Areas
At least one pair of opposite sides parallel	Perimeters
No right angles	Different shapes
Same height	Number of pairs of parallel lines

Prerequisites:
All students should have:

1. Knowledge of basic shapes and their properties.
2. Experience in comparing and contrasting.

Problem 3:
Process Standards: 1.5, 1.6, 3.3, and 3.5

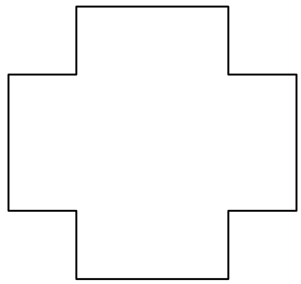
Using the two similar figures below, identify the sides from the larger triangle which correspond to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. Then identify which angles correspond to $\angle A$, $\angle B$, and $\angle C$. Justify your selections.



Solution notes:
$\overline{AB} \cong \overline{PT}, \overline{BC} \cong \overline{TF}, \overline{CA} \cong \overline{FP}, \angle A \cong \angle P, \angle B \cong \angle T, \angle C \cong \angle F$. Justifications may vary but should have some defense of how they knew which angles were congruent including a use of folding paper over to match up congruent angles and a stated understanding that corresponding sides should have a constant multiplier (each side is a set proportion larger).

<u>Problem 4</u>:
Process Standards: 1.5, 1.6, 3.3, and 3.5

Given the following figure, identify all the lines of symmetry.



Solution notes:
Students should identify four lines of symmetry. One vertical and one horizontal each splitting the figure in half, then two diagonals again splitting the figure in half.

Prerequisites:
All students should have:
1. An understanding of similar figures.
2. Corresponding sides and angles
3. Lines of symmetry

<u>Problem 5</u>:
Process Standards: 3.3 and 3.5

How many triangles can one construct with integral sides adding to 15?

Solution notes:
Students should use the Triangle Inequality to test different portions of 15.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: D
Use geometry to describe their world.

<u>Problem 1</u>:
Process Standard: 1.6

Go outside and find examples of basic geometric shapes to illustrate concepts. Find an example of:

- A point
- A pentagon
- A sphere
- Parallel lines
- Complementary angles
- Skew lines

Use the chart to record your findings.

To Find	Sketch Real World Object	Location	Clarifications Using Definitions and Terminology
Point			
Parallel lines			
Complementary Angles			

Solution Notes:
Examples:
Sphere - top of a water tower, Parallel lines - a freeway, etc.

Prerequisites:
All students should:

1. Know geometric terms and definitions.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

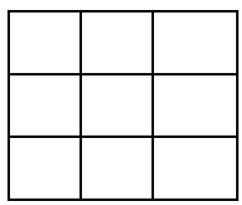
Written Benchmark: E

Extend their understanding of the process and structure for measurement.

<u>Problem 1</u>:
Process Standards: 2.1 and 3.3

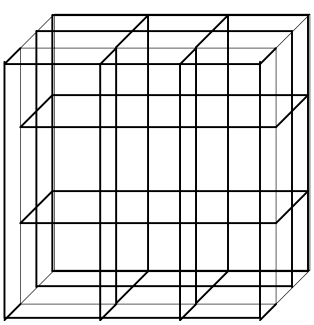
Determine the number of square feet in a square yard. Determine the number of cubic feet in a cubic yard. Explain your answer using words, mathematical sentences (computations), and diagrams.

Solution Notes:



1 yd   3 feet * 3 feet = 9 sq feet

1 yd
1 yd = 3 feet

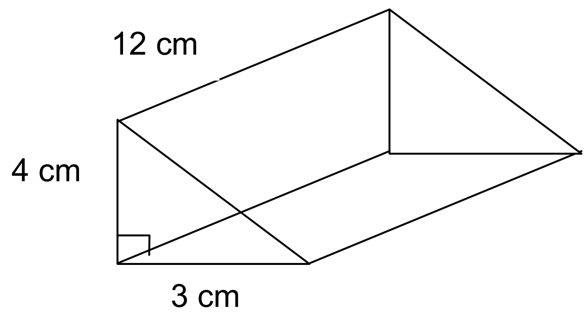


3 ft. * 3 ft. * 3 ft. = 27 cubic feet or there are 3 layers with 9 cubes in each layer.

Prerequisites:
All students should:
1. Know how to convert feet to yards.
2. Understand how to determine area and volume.

Problem 2:
Process Standards: 1.6 and 3.3

Your study pillow needs a new fabric cover. It is the shape of a regular triangular prism. Find the surface area of the pillow to determine the amount of fabric needed to cover the pillow. After calculating the amount of fabric, add on 50 square centimeters to allow for seams. Justify your calculations.



Solution Notes:

Triangle area = ½ 4cm * 3 cm = 6 cm²
x 2
Two Triangles - each end 12 cm²

*Since the bases are right triangles and the two legs are 3 and 4 cm, I know from the Pythagorean theorem that the third side must be 5 cm.

Area of Rectangle 1 → 60 cm² = 5cm X 12 cm

Area of Rectangle 2 → 36 cm² = 3cm X 12 cm

Area of Rectangle 3 → 48 cm² = 4cm X 12 cm

156 cm²
+ 50 cm² for seams
206 cm² of materials is needed to cover the pillow.

Prerequisites:

All students should know how to calculate:

1. The area of a triangle and a rectangle.
2. Surface area.
3. The Pythagorean Theorem.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: F

Select and discuss appropriate units and devices to estimate or make measurements, considering degree of accuracy.

<u>Problem 1</u>:

Process Standards: 3.3, 3.5, and 3.8

A grain farmer is going to seal the edge of the base of a circular steel grain bin with some strong seal tape. If the radius of the base of the bin is 12.5 feet, find the amount of seal tape needed.

Extension I:
The cost of the seal tape is $0.85 per foot. Find the total cost of the seal tape needed.

Extension II:
The farmer is going to cover the entire base with sealer paint, find how much area is to be covered. If sealer paint comes in 1-gallon buckets and each gallon will cover 300 square feet, how many buckets of sealer paint are needed?

Solution Notes:
Problem 1: Using 3.14 for Л → d = 25 ft. C = 3.14 (25) = C ≈ 78.5 ft. Using the Л key on the calculator, Л * 25 ≈ 78.5398 ft. = C ≈ 78.5 ft.

Extension 1:
Amount of tape needed x $0.85 per foot → $66.759 → $66.76

Extension 2:
Amount of area to be covered: Using 3.14 for Л or the Л key on a calculator, area = 490.625 ft^2 or 490.8738 ft^2. The number of buckets of sealer paint needed is 2 since 491 ft^2 / 300 ft^2 is more than one bucket of sealer paint.

Prerequisites:
All students should know how to:

1. Find the circumference and area of a circle.
2. Solve problems involving costs.

<u>Problem 2</u>:
Process Standards*:* 1.5 and 3.5

Create a circle with the diameter of 7 cm. (centimeters), using only a ruler and a compass.

Solution Notes:
The student must have the diameter of their circle within ½ centimeter of 7 centimeters.

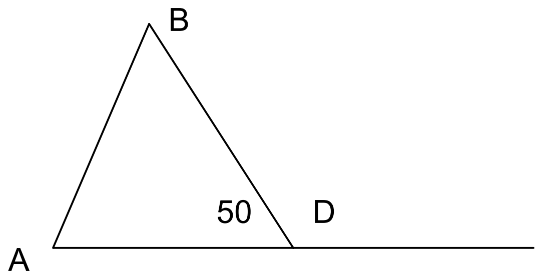
Prerequisites:
All students should:

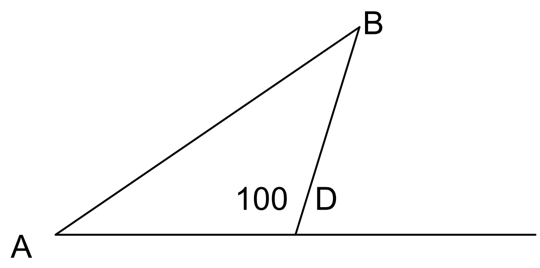
1. Know the difference between centimeters and inches.
2. Know the difference between diameter and radius.
3. Have a working knowledge of a compass and ruler.

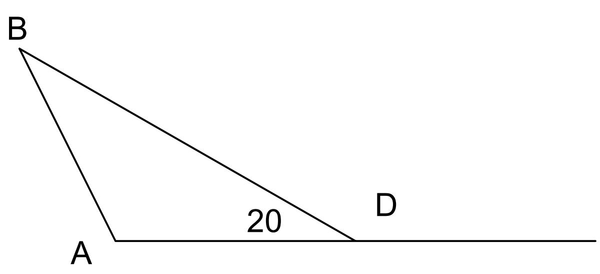
<u>Problem 3</u>:
Process Standards: 1.5 and 3.5

Given the diagram and information shown below, use a protractor to find the measure of angles A, B, and D for each triangle.

1. Describe any patterns observed between the three angles.
2. Should this pattern always work? Explain your reasoning.







Solution notes:

While reproducing these examples may change the measures of A and B, the student should realize that the sum of A and B must be the same as the measure of angle D. This can be shown both logically using geometric properties or algebraically. Both are important connections for students to experience.

Prerequisites:

All students should:

1. Know how to use the protractor to find angle measures.
2. Know that all triangles contain 180 degrees.
3. Be able to identify and express pattern recognition.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: G

Apply the concepts of perimeter, area, volume, angle measure, capacity, weight and mass.

<u>Problem 1</u>:

Process Standards: 3.3, 3.7, and 4.1

Design two rectangular gardens of different dimensions. Each garden must have an area of 24 yd^2. Use the scale 1 cm. = 2 yd. Describe the perimeter of each, then discuss the differences between the two gardens' perimeters.

Extension:

Use any computer graphics program to make the two gardens.

Solution Notes:

Any two different rectangles drawn 12 cm X ½ cm, 6 cm X 1 cm, 4 cm X 1 ½ cm, or 3 cm X 2 cm, etc.

Prerequisites:

All students should:

1. Have knowledge of conversions.
2. Have a working knowledge in the uses of a ruler.
3. Understand the meaning of area and are able to find the area of rectangles.
4. Be able to use proportions in making scale drawings.

<u>Problem 2</u>:

Process Standards: 3.3, 3.7, and 4.1

A 72-passenger school bus travels over a bridge each day on its way to school. A new sign was posted on the bridge warning 5-ton limit. An empty school bus weighs 3 tons. The average bus driver weighs 150 lbs. If the bus, filled to capacity, carries middle school students weighing an average of 106 lbs, should the bus continue to cross the bridge? Explain your answer, and include the math you used to make your decision.

Solution Notes:
No.
3 tons (bus) 7500 # (estimation of students' weight 72 * 106 lbs. is more than 7500 lb.)
7500 ÷ 2000 lb. per ton = 3.8 ton; convert the over 3 tons to 4 tons;
3 tons + 4 tons = 7 tons for school bus and passengers; too heavy for a 5 ton bridge.

The school bus should not cross the bridge.

Prerequisites:
All students should:

1. Calculate problems involving multiplication.
2. Know that 2,000lb. is one ton.

Problem 3:

Process Standards: 3.3, 3.7, and 4.1
One sports manufacturer sells their tennis balls in a cylinder can that holds exactly three tennis balls. Assuming that a tennis ball is 2.5 inches in diameter, and that the container holds three balls with no room to spare, which dimension is larger, the height or the circumference? Explain your solution.

Solution notes:
The height should only be about 3 times the diameter or 2.5 since the can holds only three balls, yet the circumference should be greater because it is $2\Pi r$, 2r is the same as 2.5 and pi is greater than 3, so circumference will be larger.

Prerequisites:
All students should:

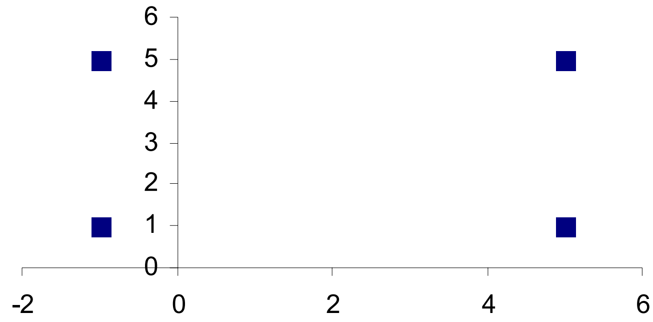
1. Know about circumference of a circle.
2. Know about volume of cylinders.
3. Be able to solve problems using 3 dimensional shapes.

Problem 4:

Process Standards: 3.3, 3.7, and 4.1

Plot the following points on a Cartesian plane (-1, 5), (5, 5), (5, 1), (-1, 1). Once the points are plotted connect them making a 4-sided figure. Find the perimeter and area for the quadrilateral.

Solution notes:
Perimeter should be 20 units and the area is 24 square units.



Prerequisites:
All students should:

1. Know and be able to calculate perimeter and area in a coordinate system.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8
Written Benchmark: H
Investigate the concept of rate of change.

<u>Problem 1</u>:
Process Standards: 3.3, 3.5, and 3.8

In 1950, the cost for a 10-ounce bag of potato chips was $0.50. In 1998, the same brand of potato chips costs $1.92 for a 10-ounce bag. Calculate the rate of change in price by calculating the price per ounce.

Extension:
Find the percent increase in the cost. At this rate predict the cost of a 10-ounce bag of potato chips in 2046.

Solution Notes:
Find the unit cost for each year:

1950	10 oz = $0.50	1 oz = $0.05	=5¢
1998	10 oz = $1.92	1 oz = $0.192	=19.2 ¢

Change of cost: 19.2¢ - 5¢ = 14.2 ¢ Number of Years: 1998 - 1950 = 48 years.

Therefore the rate of change is $\frac{14.2¢}{48 \text{ years}}$ or approximately 0.3¢ per year for one oz.

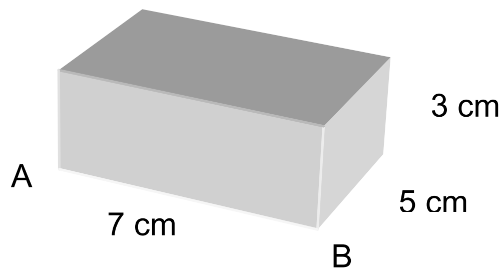
Extension Solution:
Percent increase = $\frac{\text{Amount of change}}{\text{Original Amount}} = \frac{14.2¢}{5¢} = 2.84 = 284\%$ increase in cost

Prediction for the cost of a 10-ounce bag of potato chips in 2046

$1.92 + $1.42 = $3.34

Prerequisites:
All students should:
1. Have knowledge of rates, ratios, and unit costs.
2. Know how to determine the percent of increase (extension).

Problem 2:
Process Standard: 1.6



Determine the change in the volume of a rectangular prism as the length of edge AB is increased in increments of 1 cm. Complete the table labeling your answers. Describe how and why the change in the side affects the volume.

Length of edge AB	Volume
7 cm	
8 cm	
9 cm	
10 cm	
11 cm	
12 cm	

Solution Notes:

Length of edge AB	Volume
7 cm	105 cubic cm
8 cm	120 cubic cm
9 cm	135 cubic cm
10 cm	150 cubic cm
11 cm	165 cubic cm
12 cm	180 cubic cm

Answers may vary, but should include a discussion that increasing 1-dimension impacts the whole volume because of effect of the other two dimensions.
$(7+1)\times 3\times 5=(7+1)\times 15=105+15=120$ or one more unit in length results in an additional width and depth factor.

Prerequisites:
All students should know:
1. How to determine volume of rectangular solids.

Problem 3:
Process Standards: 3.3 and 3.5

A swimmer can cross a 200-meter cove in 8 minutes. Determine the swimmer's rate in meters per second. Describe how this solution could be used to estimate how long it would take this swimmer to go 700 meters.

Solution Notes:

$$8\ \cancel{\text{minutes}} \times \frac{60\ \text{seconds}}{\cancel{\text{minutes}}} = 480\ \text{seconds}$$

$$\frac{200\ \text{meters}}{480\ \text{seconds}}$$

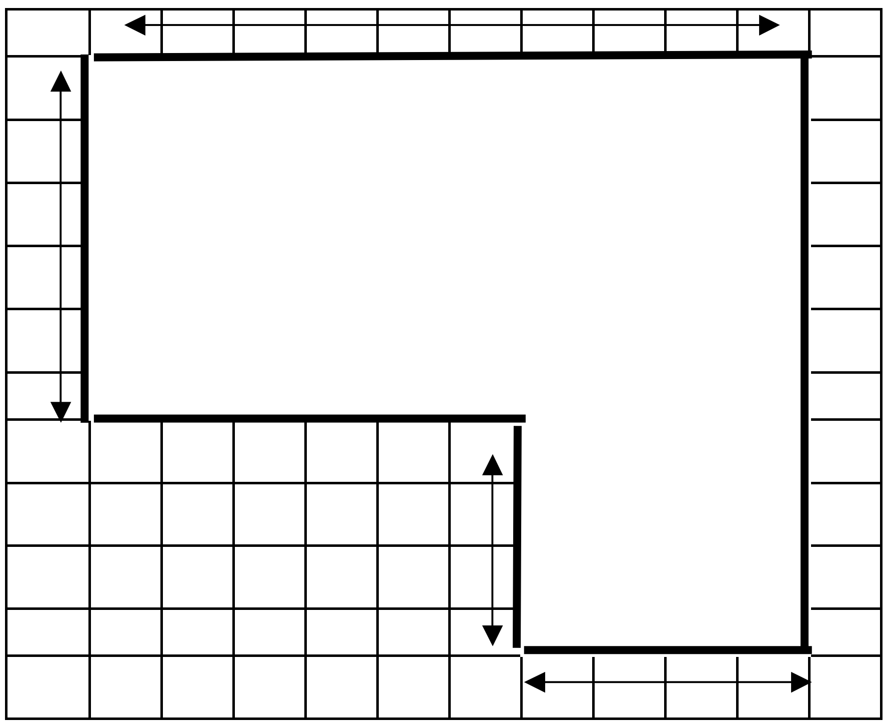
Accept any of the following: .416 m / sec. ; .417 m / sec. ; .41 2/3 m / sec.

Prerequisites:
All students should understand the meaning of:
1. Ratios.
2. Rates.

Geometric and Spatial Sense
What All Students "Should Do"
5 – 8

Written Benchmark: I
Develop formulas and procedures for determining measures to solve problems.

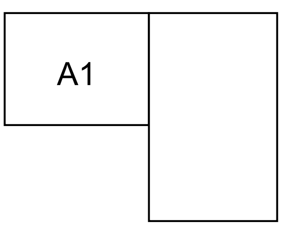
<u>Problem 1</u>:
Process Standards: 1.6 and 3.1



Find the area of the irregularly shaped figure shown as the blank region above (Each section of the grid is a 1 ft. by 1 ft. square).

Prompt:
Find the perimeter of the shape. Explain how you arrived at your answer for both perimeter and area. Be sure to label your answers. (Teacher note: This problem could be given without the grids, but with the units of length for some of the sides.)

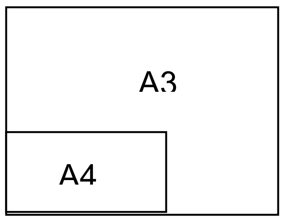
Solution Notes:
Method 1:



A1 = 6 * 6 = 36 sq. ft.
A2 = 10 * 4 = 40 sq. ft.
A1 + A2 = Total Area
Total Area = 76 sq. ft.

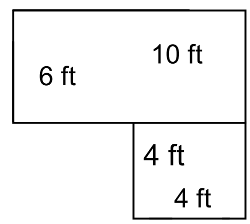
A2

Method 2:



A3 = Area of rectangle - A4
A of rectangle = 10 * 10
A of rectangle = 100 sq. ft.
A4 = 6 * 4
A4 = 24 sq. ft.
A3 = 76 sq. ft.

Solution Notes:
Solution 1:
Perimeter is 40 feet. I counted the edges and sides of the boxes around the shape.
Area = 76 square feet. I counted the squares.



Solution 2:
Perimeter = 6 + 10 + 10 + 4 + 4 + 6 = 40 feet
I added the lengths of all the sides together.
Area = 6 X 10 = 60 square feet
4 X 4 = 16 square feet
Area = 60 square feet + 16 square feet = 76 square feet
I found the areas of the two smaller rectangles and added then together.

Prerequisites:
All students should know:

1. How to determine the area and perimeter for rectangles.
2. Know how to label linear and square units.
3. Know the properties of rectangles.

DATA ANALYSIS, PROBABILITY AND STATISTICS

5 – 8

Data Analysis, Probability and Statistics
What All Students "Should Know"
5 – 8

1. Standard measures of central tendency.

Clarifications:

All students should know:

- The mean is the arithmetic average of a set of numbers.

$$\frac{\text{SUM OF THE SET OF NUMBERS}}{\text{THE NUMBER OF NUMBERS IN THE SET}}$$

- The median is the middle number in a set of number arranged in order by size, if the number of elements in the set is odd. If the number of elements in the set is even, the arithmetic average of the two middle elements in the set when the numbers in the set are arranged in order of size is the median.
  Examples:
  - ♦ Set of data with an odd number of elements: 15, 25, 75, 20, 45, 50, 35
    15, 20, 25, 35, 45, 50, 75 – arranged in order of size.
    ↑ 35 is the middle number and the median for the set.
  - ♦ Set of data with an even number of elements: 98, 95, 75, 43, 69, 73
    43, 69, 73, 75, 95, 98 – arranged in order of size.
    73 and 75 are the two middle numbers so the median is $\frac{73+75}{2}$ = 74.
- The mode is the number that occurs most often in a set of numbers.
  Examples:
  - ♦ Set of data with one mode: {25, 42, 25, 25, 63, 51,63}
    25 is the mode. It occurs three times in the set while the other numbers occur one or two times.
  - ♦ Set of data with more than one mode: {36, 42, 51, 42, 51, 64, 64}
    42, 51, and 64 are all modes since they each occur two times and that is the most any number from the set occurs.

- Range is defined in two different ways in different situations:
  - ♦ As the difference between the largest and smallest elements of the set.
  - ♦ By listing the number representing the smallest element of the set and the largest element of the set with a dash mark between the numbers.

2. Methods to analyze data.

Clarifications:
All students should know:

- The appropriate use of mean, median and mode.
- How to detect misleading data.
- How to detect extra or missing data.
- How and when to use matrices.
- How and when to use diagrams (e.g. tree).
- How to understand data in graphical or tabular forms.

3. Methods of representing analyzed data.

Clarifications:
All students should know methods for making and interpreting:

- Box - and - whiskers (box plots)
- Stem - and - leaf plots
- Frequency charts
- Scatter plots
- Multiple line graphs
- Bar graphs or Histograms
- Pie charts
- Pictographs
- Charts / tables (student constructed)
- Matrices
- Tree diagrams
- Student constructed diagrams (including Venn Diagrams)

4. Similarities and differences in theoretical and experimental probabilities.

Clarifications:
All students should know:

- Theoretical probability is the number of favorable outcomes divided by the number of possible outcomes –

$$\frac{\text{NUMBER OF FAVORABLE OUTCOMES}}{\text{NUMBER OF POSSIBLE OUTCOMES}}$$

- Experimental probability is an estimated probability determined by repeating an experiment a number of times and observing the results–

$$\frac{\text{NUMBER OF TIMES A CERTAIN RESULT OCCURS}}{\text{NUMBER OF TIMES THE EXPERIMENT IS PERFORMED}}$$

- How experimental probability relates theoretical probability with large sets of data.

5. The appropriate use of technology.

Clarifications:
All students should have available technology to use as valuable mathematical tools including:
- Computers.
- Graphing calculators.

All students should know how to use:
- Spreadsheets.
- Graphing functions on a computer.
- Four function calculators, scientific calculators and graphing calculators.

All students should use the tools of technology to:
- Explore concepts.
- Validate conjectures.
- Manipulate large numbers.
- Analyze large sets of data.
- Graph equations and data.
- Solve problems.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: A

Develop, analyze, and explain methods utilized
to collect, organize, and describe data.

<u>Problem 1</u>:

Process Standards: 1.4 and 2.1

Write a paragraph describing whom you would choose to take the Lung Capacity Test (how long a person can hold their breath) and explain why you chose that person.

Solution Notes:
The sample should include a variety of sports, musicians, and activities in which the student is interested in or is involved in as a participant. The explanation should reinforce their choices and make use of data.

Prerequisites:
All students should know:

1. The techniques of sampling.

<u>Problem 2</u>:

Process Standards: 1.1 and 2.1

Create a questionnaire that includes at least 5 questions to ask students to compare athletes' and musicians' lung capacity (Lung Capacity Survey).

Solution Notes:
Make sure questions are relevant to lung capacity and the sample includes a variety of athletes and musicians (choir, band, smoker vs. non-smoker etc.).

Prerequisites:
All students should know:

1. How to use the techniques of sampling.
2. How to write good questions.
3. How to test your questions, then edit/revise for clarification or improvement.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: B

Make, read, and interpret multiple representations
including tables, charts, and graphs of data.

<u>Problem 1</u>:

Process Standards: 1.5 and 3.3

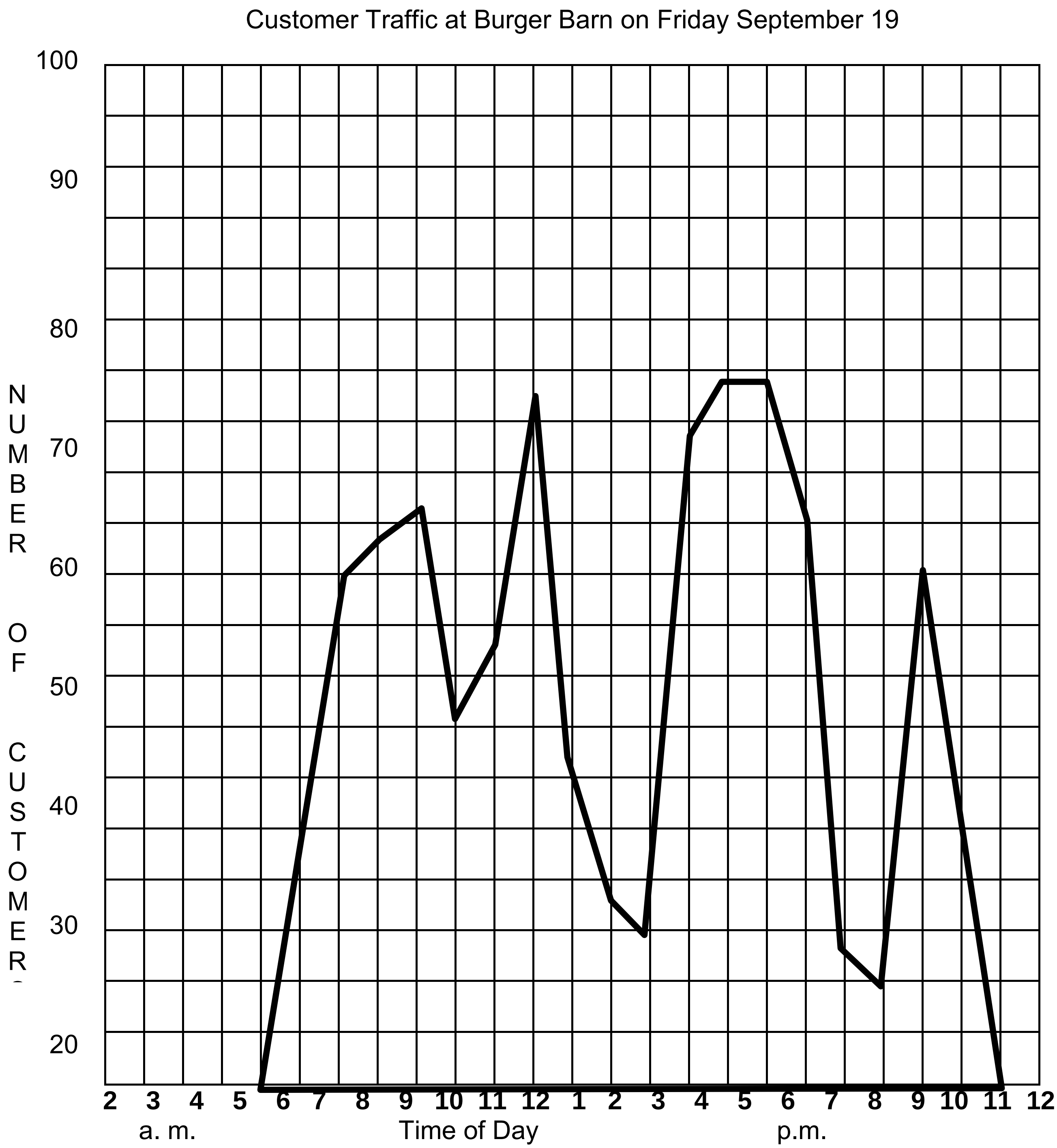
The manager of Burger Barn was interested in the restaurant's flow of customers on a typical day. The data he collected are represented in the graph on the next page. Using the graph, choose 5 specific one-hour time ranges, and then write a reasonable explanation for each change in the graph.

Solution Notes:
Students must pick 5 specific times illustrated on the graph and then provide reasonable explanations for the surge or decline in customers.

Example:
9 p.m. there is an increase due to the Friday night football game ending just after 8 p.m.

Prerequisites:
All students should:

1. Know how to read and interpret graphs.


Customer Traffic at Burger Barn on Friday September 19
100
90
80
70
60
50
40
30
20
NUMBER OF CUSTOMERS
2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
a. m.
Time of Day
p.m.

Problem 2:
Process Standard: 1.8

Using a stopwatch or watch with a second hand, time how long students can hold their breath to the nearest second. After creating a classroom data list, display the data in 3 different visual representations. Compare the strengths and weaknesses of each of your representations.

Solution Notes:
Options could include:
- Stem and leaf plot
- Box and whisker plot
- Frequency chart
- Bar graph
- Other representations if they give a good visual model of the data

Prerequisites:
All students should:
1. Have knowledge of different types of visual representations.

Problem 3:
Process Standard: 1.8

A company hires men and women for jobs in three categories: executive, supervisor, and worker. The company claims that it is not discriminating against female applicants, since it hires about the same overall proportions of men and women. (130/800 men compared to 135/800 women). A women's group claims that the company is discriminating against female applicants.

A. Fill in the second table to obtain a different perspective.
B. In this situation, which argument is more valid, that of the women's group or that of the company?

Table 1

Category	Number of Men who Applied	Number of Men Hired	Number of Women who Applied	Number of Women Hired
Executive	600	60	100	5
Supervisor	100	50	400	100
Worker	100	20	200	30
TOTAL	**800**	**130**	**800**	**135**

Table 2

Category	Percentage of Men Applicants Hired	Percentage of Women Applicants Hired
Executive		
Supervisor		
Worker		

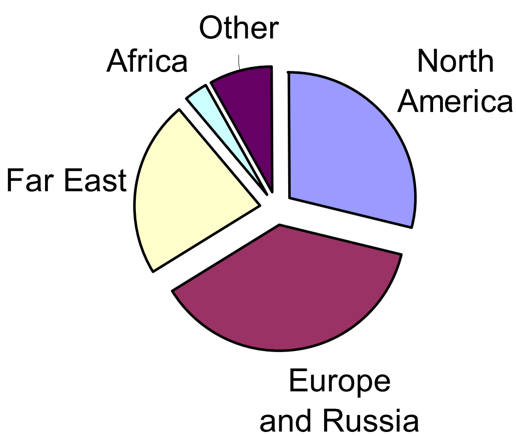
Solution notes:

A. Men: 10%, 50%, 20%; Women: 5%, 25%, 15%.
B. The women's group position is more valid. Students should explain that the percentage of applicants for each job who are actually hired is consistently lower for women than for men.

Prerequisites:
All students should know:

1. How to compute and compare percentages.

<u>Problem 4</u>:
Process Standard: 1.8
Estimate the percent of energy consumption for each geographic area.



Solution Notes:
Student answers may vary. Look for reasonable relationships. For example, North America should be greater than 25%, and Europe and Russia should be bigger than North America. Far East should be less than 25%. Africa and Other together should be 10% or less. The student's percentages should add to 100%.

Exact answers are: North America 29%
Europe and Russia 37%
Far East 23%
Africa 3%
Other 8%

Prerequisites:

All students should know:

1. How to estimate pieces from a circle graph.
2. Relationships between fractions of a circle and percents (e.g., ¼ = 25 %).
3. The sum of percents in a circle graph is always 100%.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: C

Formulate, predict, and defend positions taken that are based on data collected.

Problem 1:

Process Standards: 1.2 and 1.4

As a class create a chart by measuring the neck and wrist circumference of each class member. From the data gathered, predict the smallest watch band (length) you could purchase given a neck measurement.

Solution Notes:

The ratio should be about 2: 1 (neck to wrist). Students may arrive at the ratio comparison of the 2 measurements by making a scatter plot, line of best fit, finding the mean, etc.

Prerequisites:

All students should know:

1. The meaning of ratio.
2. How to estimate.
3. How to measure.
4. How to determine the mean of a set of data.
5. How to determine the line of best fit.

Problem 2:

Process Standards: 1.6, 3.4, and 3.7

Given the following test scores of 100, 68, 89, and 93 determine what you need to score on the next test to have a mean score of 90. Is this score possible? Why is it possible or why is it not possible? How will the range be affected? Show your work mathematically.

Solution Notes:

$\frac{X + 350}{5} = 90$; the solution is a score of 100. The range is not affected. If each test is based on 100, then this result is possible.

Prerequisite:

All students need to know:

1. How to determine the mean and range of a set of data.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: D

Analyze information and arguments that
are based on data collected.

<u>Problem 1</u>:
Process Standards: 3.5 and 3.6

Analyze the chart; draw conclusions describing the weather for each day of the week. Justify your argument with data from the chart and explain your decision in paragraph form. Display your conclusions in a chart.

Attendance at Outdoor City Pool

Day of the Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of People	30	75	85	91	0	45	80

Solution Notes:
Considerations:

- Rain
- Temperature
- Lighting
- Predicted temperature
- Very cold
- Holidays

Prerequisites:
All students should:

1. Be able to construct and interpret charts.

<u>Problem 2</u>:
Process Standards: 1.7 and 3.6

Our class conducted a survey and reported the number of pets per student to be 3.47. Could this number represent the:

- Mean?
- Median?
- Mode?
- Range?

Explain your reasoning for each.

Solution Notes:
Could this number represent the:

- Mean? Yes because average could be a decimal.
- Median? No, the median must be 0.5 or a whole number for this situation since it is not possible to have only a part of a pet.
- Mode? No, the mode would be a whole number because the numbers in the data set represented each student's number of pets which are all whole numbers.
- Range? No, the range is whole numbers only.

Prerequisites:
All students should be able to determine the:

1. Mean for a set of data and interpret the meaning of this measure of central tendency.
2. Median for a set of data and interpret the meaning of this measure of central tendency.
3. Mode for a set of data and interpret the meaning of this measure of central tendency.
4. Be able to determine the range for a set of data and interpret the meaning of this measure of variability.

<u>Problem 3</u>:
Process Standards: 1.7 and 3.6

On their most recent test, student scores were: 39, 39, 39, 40, 43, 44, 45, 47, 47, 48, 49, 50, 50, 51, 99, 100. Compute the mean, median, and mode for this group of numbers. Which measure of central tendency best describes the data? Tell why.

Solution notes: Mean = 51.7, Median = 47, Mode = 39.
The median is the best measure of central tendency. Students should include comments about the outliers' (99 and 100) effect on the mean – making it too high, and the mode being unrealistic because the most frequent number was on the outer portion of the data set. Mode scores are most appropriate when the data set is very large.

Prerequisites:

All students should be able to determine the:

1. Mean for a set of data and interpret the meaning of this measure of central tendency.
2. Median for a set of data and interpret the meaning of this measure of central tendency.
3. Mode for a set of data and interpret the meaning of this measure of central tendency.
4. Be able to determine the range for a set of data and interpret the meaning of this measure of variability.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: E

Investigate the power of making decisions based on statistical methods and the applications of probability in the real world.

<u>Problem 1</u>:

Process Standards: 3.2 and 4.7

If the meteorologist predicts a 75% chance of heavy snow on Wednesday, how would this forecast affect your plans? List 5 things that could be affected by the forecast. Justify each decision.

Solution Notes:
Make sure the ideas are related to weather and the chance of snow.

Prerequisites:
All students should:

1. Have knowledge of probability.

<u>Problem 2</u>:

Process Standards: 3.2 and 4.3

The meteorologist forecasts a 75% chance of snow at a resort you are going to on Friday. It is about 150 miles away. You plan on staying Friday, Saturday and coming home on Sunday. List 5 things that may be affected by the forecast and justify each decision.

Solution Notes:
Make sure the ideas are related to weather.
Example: I will need …

Prerequisites:
All students should:

1. Understand the meaning of probability.

<u>Problem 3</u>:

Process Standards: 3.2 and 4.3

Describe how these two situations (Problem 1 and Problem 2) relate to one another. Explain your reasoning.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: F

Use computers, graphing calculators, and/or other forms of technology to enhance understanding of numbers, data, and the resulting analysis.

<u>Problem 1</u>:

Process Standards: 1.4 and 2.7

Enter the given data on a spreadsheet on the computer (or list on a graphing calculator). Create a graph (i.e. pie chart, histogram) including labels and percent of each item in the given data.

Solution Notes:
Provide data for the student to use to make the graph or let the students generate data to enter into the computer to make a graph.

Prerequisites:
All students should be able to use:

1. Spreadsheet program.
2. Computer graphics.
3. A graphing calculator to generate a graph to represent the data.

<u>Problem 2</u>:

Process Standards: 1.4 and 2.7

Enter the data on a spreadsheet on the computer (or list on a graphing calculator). Use the functions on the spreadsheet to determine the mean, median, mode, and range.

Solution Notes:
Provide students with data or have students collect data to enter into a computer spreadsheet or a graphing calculator.

Prerequisites:
All students should know how to:

1. Use a computer spreadsheet.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: G

Develop and execute experiments or simulations
to predict and determine probable outcomes.

Problem 1:

Process Standards: 1.6 and 3.6

Your teacher has decided to have a pizza party for the class. You would like to have a hand-tossed, extra cheese pepperoni pizza. List all the probable combinations of pizza your teacher can order. Determine the probability of your getting the pizza of your choice (assuming each combination is equally likely).

Crust:	hand-tossed, thick, thin
Toppings:	pepperoni, hamburger, sausage, ham, shrimp
Cheese:	regular, extra-cheese

If you choose to abbreviate your list of probable combinations, make sure you provide a key.

Solution Notes:

A tree diagram will be helpful, but is not required. If you wish to use abbreviations, provide a key.

Sample Key:

TK = Thick Crust	EC = Extra Cheese	HB = Hamburger
HT = Hand Tossed	Rg = Regular Cheese	SA = Sausage
TH = Thin Crust		HM = Ham
		PP = Pepperoni
		SH = Shrimp

Probable Combinations:

TK, HB, EC	HT, HB, EC	TH, HB, EC	TK, HM, Rg	HT, HM, Rg	TH, HM, Rg
TK, HB, Rg	HT, HB, Rg	TH, HB, Rg	TK, PP, EC	HT, PP, EC	TH, PP, EC
TK, SA, EC	HT, SA, EC	TH, SA, EC	TK, PP, Rg	HT, PP, Rg	TH, PP, Rg
TK, SA, Rg	HT, SA, Rg	TH, SA, Rg	TK, SH, EC	HT, SH, EC	TH, SH, EC
TK, HM, EC	HT, HM, EC	TH, HM, EC	TK, SH, Rg	HT, SH, Rg	TH, SH, Rg

The probability of getting a hand-tossed, extra cheese pepperoni is 1/30.

Prerequisites:

All students should know how to:

1. Make a tree diagram and have knowledge of probability.

Problem 2:
Process Standards: 3.1 and 3.6

Given a 16-word sentence, “the” was used three times. In a paragraph consisting of 60 words, use probability to determine the number of times the word “the” should occur.

Solution Notes:
The answer must be rounded to a whole number since words would be in whole numbers.

$$\frac{3}{16} = \frac{X}{60}$$

$$16X = 3 * 60$$

$$16X = 180$$

$$X = 11.25$$

X = 11, so 11 words out of 60 would be the word ”the” based on the probability given in the problem.

Prerequisites:
All students should:
1. Be able to determine probability for a given situation.

Problem 3:
Process Standards: 3.1 and 3.6

The island nation Walnut is issuing license plates for the first time. If they plan to have license plates with 2 letters and 2 digits on them, how many different plates can they make? Will this provide enough plates for their entire population of 23,071?
[Sample plate might be RB53]

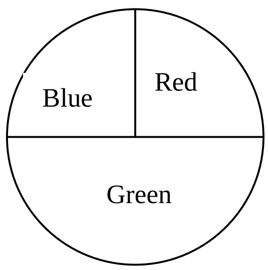
Solution notes:
67,600. Yes, unless the population triples or everyone owns multiple cars.

Prerequisites:
All students should:
1. Be able to determine number of combinations for a given situation.

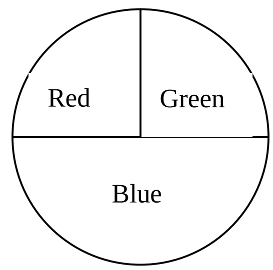
Problem 4:

Process Standards: 3.1 and 3.6

Spinner 1

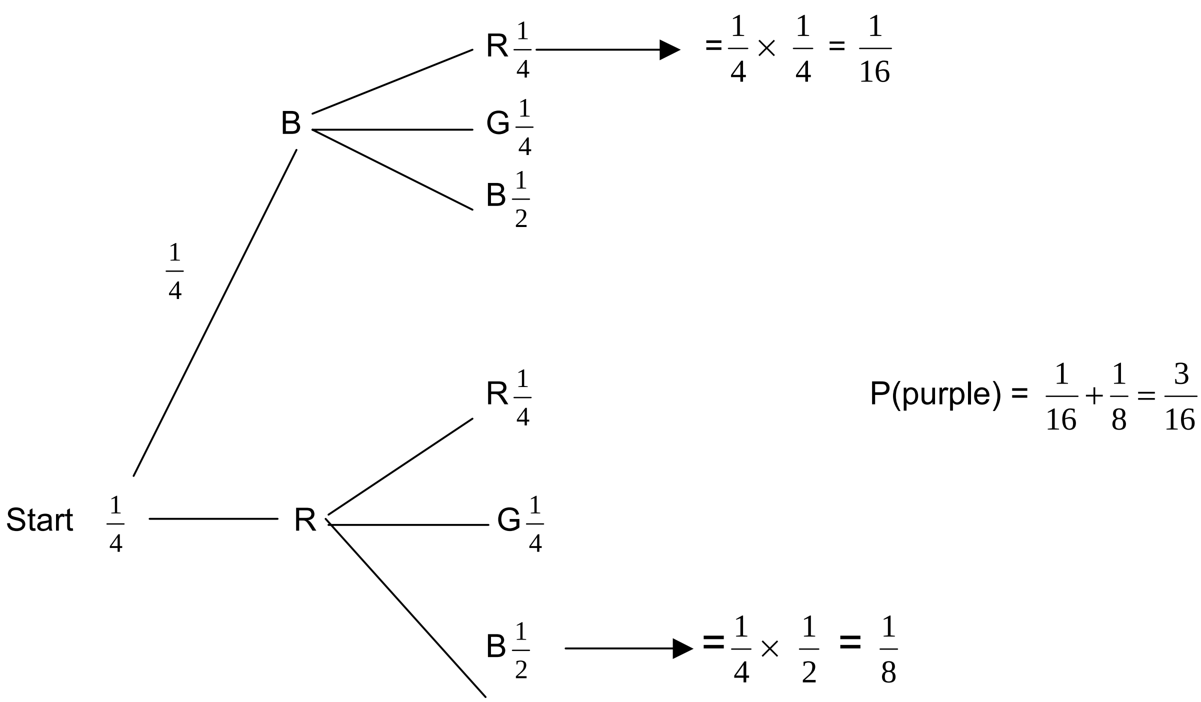


Spinner 2



If you spin each spinner once, what is the probability of getting “purple”? (i.e., one red and one blue)?

Solution notes:



Prerequisites:

All students should:

1. Be able to calculate the probability for a given situation.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: H

Investigate sample spaces to predict probable outcomes and how these predictions affect the decision-making process.

Problem 1:

Process Standards: 1.2, and 1.10

Given a bag of 10 items, students will draw one item from the bag without looking into the bag. Once the item is drawn, students will record the color and then put the item back in the bag. Repeat this process 30 times. Use the results to predict the number of items of each color in the bag.

Teacher Notes:

Groups of 4 students work well together. Bags are prepared ahead of time so that students do not know the contents of the bags. If a larger sample space is desired, group data can be combined; in this case all bags would have to have the same contents, otherwise results would vary. Use 4 different colors.

Solution Notes:

Answers will vary based on color arrangements of actual bag contents and experimental results.

Prerequisites:

All students should be able to:

1. Create a chart to show the totals of combining the data from the groups.

Problem 2:

Process Standards: 1.10, 3.6, and 4.1

Roll two number cubes 50 times. Add the numbers together each time. If the sum is even, you win; if the sum is odd, your partner wins. From your results determine if the game is fair or unfair. Mathematically explain (using a graph, diagram, chart, or equation) why the game is fair or why the game is not fair?

Solution Notes:
Example Solution:
The game is fair; the probability of winning if the sum is even is ½:
even + even = even
odd + odd = even
odd + even = odd
even + odd = odd
2 and 6 = 8
1 and 3 = 4
7 and 4 = 11
6 and 3 = 9

Prerequisites:
All students should know:
1. Characteristics of odd and even numbers, and their sums.
2. How to use graphs, diagrams charts or equations.
3. How to extend experimental results to make reasonable conclusions.

<u>Problem 3</u>:
Process Standards: 1.10, 3.6, and 4.1

If you roll two tetrahedral dice (dice with four sides) and multiply their outcomes, the results are shown in the table below. Find the probabilities for: P(16), P(5), P(4), and P(12).

X	**1**	**2**	**3**	**4**
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Solution notes:
P(16) = $\frac{1}{16}$, P(5) = 0 , P(4) = $\frac{3}{16}$, and P(12) = $\frac{1}{8}$.

Prerequisites:
All students should:
1. Be able to calculate the probability for a given situation using data provided.

Data Analysis, Probability and Statistics
What All Students "Should Do"
5 – 8

Written Benchmark: I

Investigate appropriate applications for
experimental and theoretical probabilities.

<u>Problem 1</u>:

Process Standards: 1.10, 1.3, and 3.3

Design a real-world experiment that demonstrates experimental probability.
One example might be: In a room with a tile floor made up of 1' by 1' squares, drop a toothpick on the floor 100 times and determine the experimental probability of the toothpick landing on a line.

Solution Notes:
Answers will vary. Make certain the explanation of experimental probability is clearly defined.

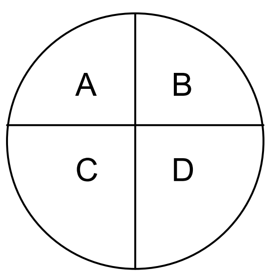
Prerequisites:
All students should:

1. Know the definition of experimental probability.

<u>Problem 2</u>:

Process Standards: 1.2 and 1.3

Use the spinner below to find the theoretical and experimental probability of spinning an A. Label your answer.



Solution Notes:
Theoretical probability is ¼. Experimental probability should show repeated trials. Answers will vary.

Prerequisites:
All students should:

1. Know how to determine theoretical probability.
2. Know how to determine experimental probability.

PATTERNS AND RELATIONSHIPS

5 – 8

Patterns and Relationships
What All Students “Should Know”
5 – 8

1. Mathematical ideas will be represented with visual models.

Clarifications:
All students should know examples of visual models representing patterns and relationships:

- Function machines (input/output)
- Tables
- Drawings of the situation
- Graphs
- Diagrams (e.g. Tree)
- Manipulative
- Number patterns illustrated by a geometric pattern

2. Mathematical symbols can be used to represent real-world situations.

Clarifications:
All students should know how to translate a problem stated in English into math symbols:

- Equations
- Inequalities ($<$, $>$, $\leq$, $\geq$, $\neq$)
- Union and intersection
- Diagram (i.e. Venn, Tree, or student generated)
- Arithmetic operations

3. Patterns and relationships can be represented in a variety of ways.

Clarifications:
All students should know ways to illustrate patterns and relationships:

- Lists
- Written rule / algebraic equation
- Function machine (input, output)
- Table
- Description (verbal and written)
- Graphs

4. Information can be organized to look for a pattern and relationship.

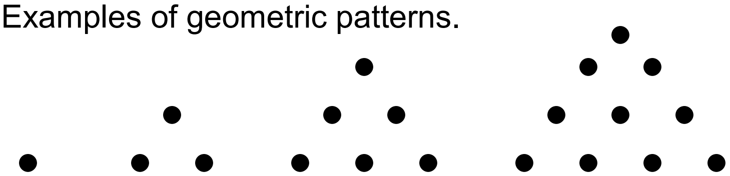
Clarifications:
All students should know ways to organize information:

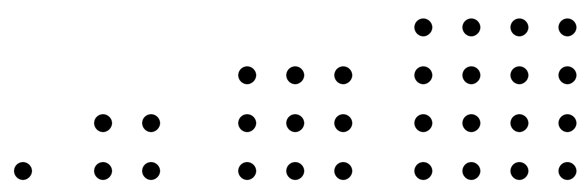
- Tables
- Graphs
    - Bar
    - Line
    - Pie
    - Scatter plot
    - Box and whiskers
    - Stem and leaf
- Pictures
- Visual patterns
- Analogies
- Spreadsheets
- Graphing equations of lines

5. Patterns can be geometric and / or numeric.

Clarifications:
All students should know:

- Examples of geometric patterns.





- Examples of numeric patterns:
    - 1, 3, 6, 10 **. . .**
    - 1, 4, 9, 16, 25 **. . .**
    - 5, 8, 11, 14 **. . .**
    - 1, 1, 2, 3, 5, 8, 13 **. . .** (Fibonnaci)

Patterns and Relationships
What All Students "Should Do"
5 – 8

Written Benchmark: A
Examine, predict, design, extend and describe patterns and relationships.

Problem 1:
Process Standards: 1.6 and 2.1

-8, -4, 0, 4, _____ , _____ , _____

Identify the next three terms in the sequence shown above. Explain how you determined your answer. Use your model to determine the 100^{th} term. Show how you used your model or explain how you determined the 100^{th} term in the pattern.

Solution Notes:
Completing the next three terms of the sequence gives the following:
-8, -4, 0, 4, 8 , 12 , 16

Explanation 1:
Add 4 every time, or 4n - 12 (n = number of terms). Explanations may vary, but as long as they generate the same results accept the explanation.

Explanation 2:
The difference between each of the terms is +4, so 4n where n = number of terms if 0 is the first term, but –8 is the first term so you would have to subtract 12 or a formula of [4n – 12].

N	Term
1	-8
2	-4
3	0
4	4
5	8
6	12

+ 4
+ 4
+ 4
+ 4
+ 4

Prerequisites:
All students should know how to:
1. Add integers.
2. Reason inductively.

Problem 2:
Process Standards: 1.5 and 1.6

How many blocks will be used to create the fourth figure in the pattern? In the box below explain how you determined the answer.

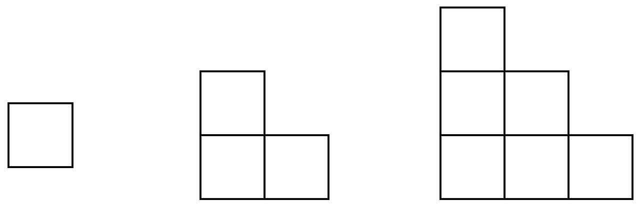


Figure 1 Figure 2 Figure 3 Figure 4

Solution Notes:
10 blocks will be in figure 4. The explanation can be by drawing the figure and counting, by aTable or written in words. To determine the number of blocks for each figure, look at the number of blocks in the previous figure and add the number to that number. Example:

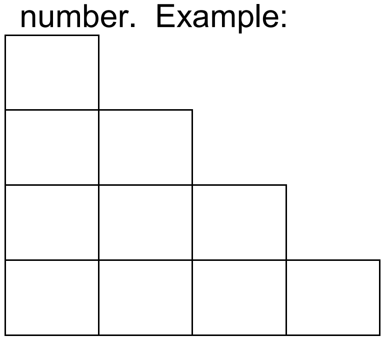


Figure #	# of Blocks
1	1
2	3
3	6

Prerequisites:
All students should:
1. Know counting techniques.
2. Be able to apply critical thinking skills.

Patterns and Relationships
What All Students "Should Do"
5 – 8

Written Benchmark: B

Design and compare patterns and relationships using rules, charts, and graphs that may be constructed using technology.

<u>Problem 1</u>:.
Process Standards: 1.6 and 3.3

Find the missing numbers in the table. Write the rule for the table.

Input	Output
2	8
4	16
6	□
□	20

Solution Notes:
Multiply the input by 4 to get the output or 4 *I*. If the input is 6, then the output is *<u>24</u>*. If the output is 20, then the input is *<u>5</u>.*

Prerequisites:
All students should:

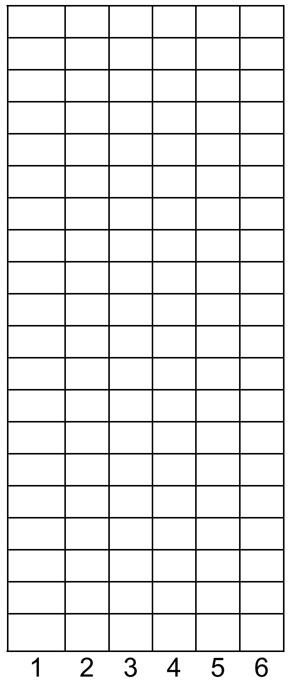
1. Know basic computational skills.
2. Be able to apply critical thinking skills.
3. Be able to recognize and use patterns generated.

<u>Problem 2</u>:
Process Standards: 1.8 and 3.5

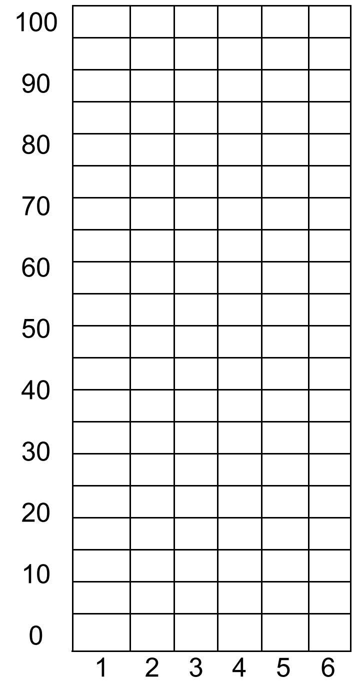
Math	70	68	65	78	72	89
History	100	92	85	83	81	80

The chart above shows a student's scores for six math and history tests. Construct a scatter plot for each set of data on the grids provided below. Construct a line of best fit for each subject. Based on the trend shown by each line of best fit, predict the next test score for each subject.

MATH



HISTORY



Solution Notes:
Math scores on the next test will be in the range 75-79 based on a prediction from the scatter plot of the past six mathematics scores. History scores on the next test will be in the range of 90 - 93 based on a prediction from the scatter plot of the past six history grades. Scatter plot and line of best fit must be constructed correctly.

Prerequisites:
All students should know how to:

1. Make a scatter plot.
2. Estimate a line of best fit.

Problem 3:
Process Standards: 1.6, 1.10, and 4.1

Marie was getting ready to tile the floor using 12-inch tiles and this pattern:

☼	□	□	□	☼	□	□	□
□	◆	◆	◆	□	◆	◆	◆
□	◆	□	◆	□	◆	□	◆
□	◆	◆	◆	□	◆	◆	◆
☼	□	□	□	☼	□	□	□
□	◆	◆	◆	□	◆	◆	◆
□	◆	□	◆	□	◆	□	◆
□	◆	◆	◆	□	◆	◆	◆

60 white tiles □ 110 blue tiles ◆ 20 yellow tiles ☼

One edge of her 21 feet X 21 feet floor will have these tiles.

☼				☼				☼				☼				☼				☼

Using the pattern shown in the rows and columns above, how many more of each kind of tile will Marie need to finish her floor? Explain how you solved the problem in the box below.

She will need:
_______ more white tiles □
_______ more blue tiles ◆
_______ more yellow tiles ☼

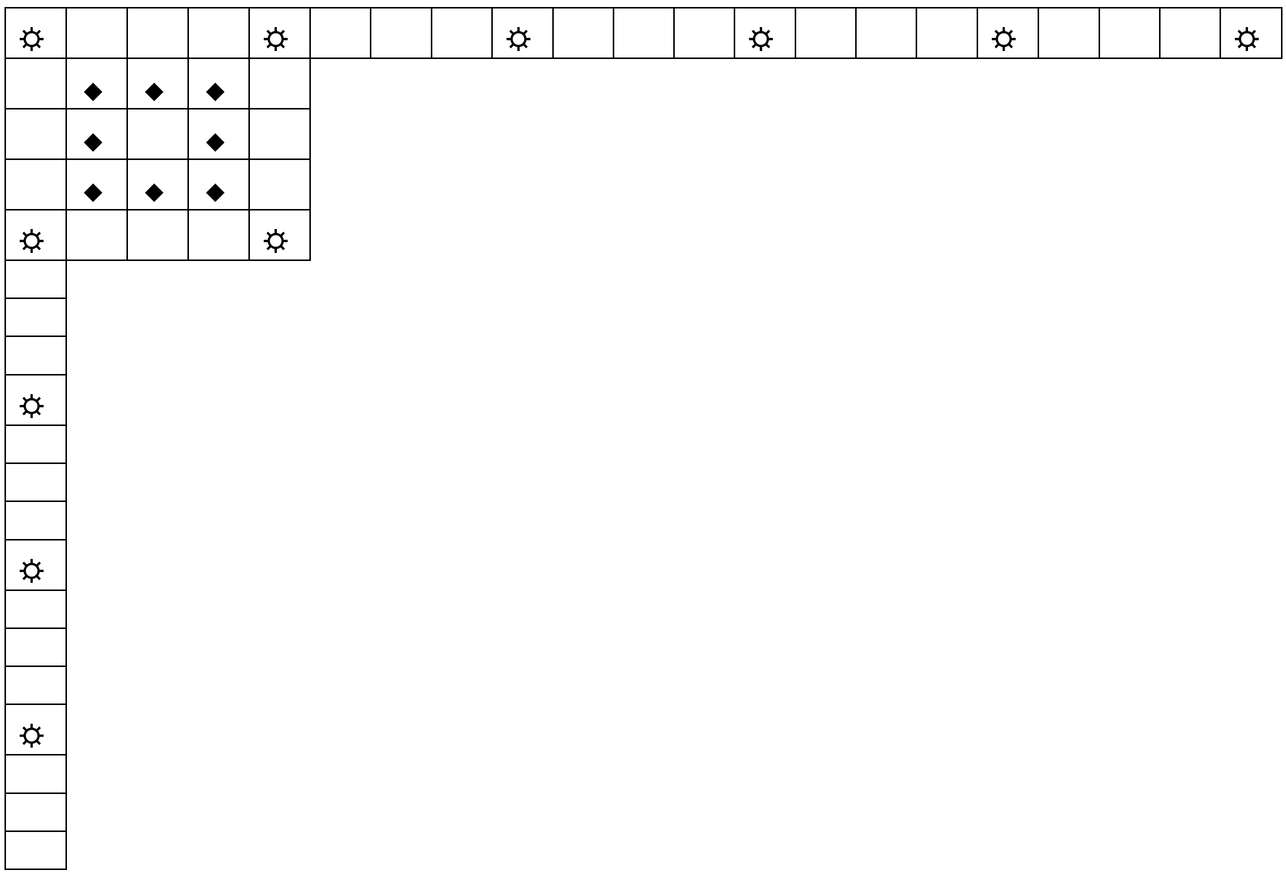
Solution Notes:

145 more white tiles □, 90 more blue tiles ◆, 16 more yellow tiles ☼

The students could use color tiles or they could draw a diagram. It is also possible to solve the problem mathematically (using arithmetic).

Mathematically: 15 rows have blue tiles; 21 rows of white tiles; 10 rows have 15 blue tiles 10 X 15 = 150; 6 rows have 15 white tiles 6 X 15 = 90; 5 rows have 10 blue tiles 5 X 10 = 50; 10 rows of 6 white tiles 6 X 10 = 60; 5 rows of 11 white tiles 5 X 11 = 55.

150 + 50 = 200 blue tiles are needed.   90 + 60 + 55 = 205 white tiles are needed.



Prerequisites:
All students should be able to:

1. Apply problem solving.
2. Use critical thinking skills.
3. Arithmetic skills.

Patterns and Relationships
What All Students “Should Do”
5 – 8

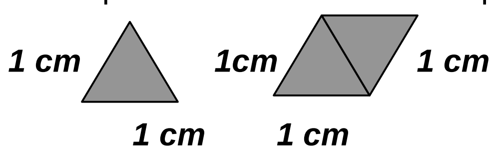
Written Benchmark: C

Examine patterns, relationships and functions to determine how a change in the independent variable can produce a change in a dependent variable.

<u>Problem 1</u>:

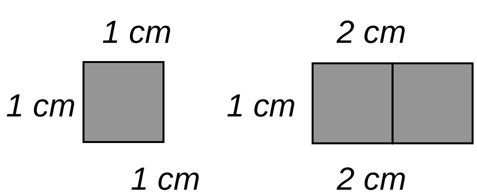
Process Standards: 1.6 and 3.4

Given the following shapes create patterns adding an additional shape for the next figure. Complete the table for each sequence.



Complete the table:

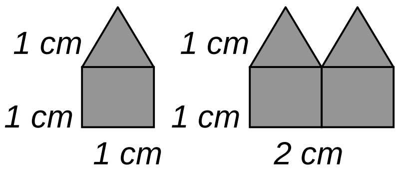
Number of Triangles	Perimeter in Centimeters
1	3
2	4
3	5
4	
5	

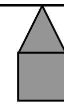


Complete the table:

Number of Squares	Perimeter in Centimeters
1	4
2	6
3	8
4	
5	

2. Describe the patterns involved in these problems, then find an algebraic equation that would produce the same values as listed in each pattern.Conduct the same experiment for a five sided figure:



Number of 	Perimeter in Centimeters
1	5
2	8
3	11
4	
5	

Complete the table:
Create the figure and complete the table for the polygon formed by connecting six-sided figures in a similar manner.

1 adjoining figure

2 adjoining figures

Complete the table:

Number of Figures	Perimeter in Centimeters
1	6
2	
3	
4	
5	

Solution Notes:
Perimeter of 4 triangles in the pattern is 6 cm. Perimeter of 5 triangles in the pattern is 7 cm. Perimeter of 4 squares in the pattern is 10 cm. Perimeter of 5 squares in the pattern is 12 cm. Perimeter of 4 houses in the pattern is 14 cm. Perimeter of 5 houses in the pattern is 17 cm. Perimeter of 2 six sided figures in the pattern is 10 cm. Perimeter of 3 six sided figures in the pattern is 14 cm. Perimeter *of* 4 six sided figures in the pattern is 18 cm. Perimeter of 5 six sided figures in the pattern is 22 cm.

Prerequisites:
All students should be able to:
1. Add.
2. Use reasoning skills.
3. Use problem-solving strategies.
4. Apply critical thinking skills.

<u>Problem 2</u>:
Process Standards: 4.2 and 1.5

In Turtleville, Missouri the speed limit is 35 miles per hour. There is a $50.00 fine for speeding plus $5.00 for every mile over the speed limit. Officer Harry clocked Mary Ann at 45 miles per hour on Monday, at 53 miles per hour on Tuesday, and at 61 miles per hour on Wednesday. What was her fine on each of these days?

Monday $__________	Tuesday $__________	Wednesday $__________

On Thursday, Mary Ann received a ticket for $200.00 for speeding. How fast was she going?

__________Miles per hour

Solution Notes:

Monday	$50.00 + (10 * $5.00) = $100.00
Tuesday	$50.00 + (18 * $5.00) = $140.00
Wednesday	$50.00 + (26 * $5.00) = $180.00
Thursday	$50.00 + [(X - 35)* $5.00] = $200.00
	$50.00 + 5X - $175 .00 = $200.00
	5X - $125.00 = $200.00
	5X = $325.00
	X = 65 miles per hour

The problem could also be worked by plotting a graph of speeds vs. fines to find the 65 miles per hour. Other methods may be possible that would generate the correct results.

Prerequisites:
All students should be able to:
1. Use problem-solving strategies.
2. Use critical thinking skills.
3. Apply computation skills to real world situations.

Patterns and Relationships
What All Students "Should Do"
5 – 8

Written Benchmark: D

Apply patterns, relations, and functions to solve real world problems.

<u>Problem 1</u>:

Process Standards: 1.6 and 3.2

The distance from St. Louis to Chicago is 330 miles. If your dad drives at an average rate of speed of 60 miles per hour, how many hours will it take to get to Chicago? Show your work below.

Solution Notes:

Method 1:
r * t = d
60 t = 330
t = 5 ½ hours or 5 hours 30 minutes
Other methods are possible as long as they generate the correct results.

Prerequisites:

All students should be able to:

1. Use problem-solving strategies.

<u>Problem 2</u>:

Process Standards: 1.6 and 3.2

Your pen pal in Australia has just written you a letter telling you the temperature on New Years Day was 40 ° C. What was the equivalent temperature in degrees Fahrenheit? Show your work below.

Solution Notes:

$F = \frac{9}{5}(40) + 32$
F = 9 · 8 + 32
F = 72 + 32
F = 104°

Prerequisites:

All students should be able to use:

1. Computation skills to solve real world situations.
2. Substitution to solve problems.

Patterns and Relationships
What All Students "Should Do"
5 – 8

Written Benchmark: E
Solve equations and inequalities.

<u>Problem 1</u>:
Process Standards: 1.6 and 1.8

Solve the following equation for x. Show each step.

2 X + 5 = 17

____
____
____
____
____
____
____

Solution Notes:

$$\begin{aligned} 2X + 5 &= 17 \\ 2X + 5 - 5 &= 17 - 5 \\ 2X &= 12 \\ \frac{2X}{2} &= \frac{12}{2} \\ X &= 6 \end{aligned}$$

or

$$\begin{aligned} 2X + 5 &= 17 \\ -5 &= -5 \\ 2X &= 12 \\ \frac{2X}{2} &= \frac{12}{2} \\ X &= 6 \end{aligned}$$

A student would receive <u>full</u> points for all proper (some combining of steps may be appropriate) steps. A student would receive some points despite some errors in the steps.

Prerequisites:
All students should:
1. Know how to use computational skills.
2. Be able to apply algebraic thinking.

Problem 2:
Process Standards: 1.6 and 1.8

Bill and Sue realized that together they had 45 CDs, and they knew Sue had 3 CDs less than twice the number that Bill owned. (Sue almost had two times more CDs than Bill.) How many CDs did each own?

Solution notes:

B + S = 45→ B + S = 45 3B = 48 or B = 16 so S = 29

S + 3 = 2B→ -2B + S = -3

Method 2:

BILL	SUE
0	45
5	40
10	35
15	30
20	25
16	29

Problem 3:
Process Standards: 1.6 and 1.8

A few weeks later (after more CDs were purchased) Bill and Sue gave their math teacher this problem to solve: (The teacher knew how many CDs they had before).

Together we have almost 51 CDs and Sue has more than twice the number of CDs that Bill has. How many CDs do each of the students have? Justify your answer.

Solution notes:

Since the teacher knows that Bill has at least 16 and Sue at least 29, these values provide starting points. Bill's number is important because it now forces the number Sue has because Sue has more than twice the number Bill owns.

Here are some possible solutions:
(CDs Bill owns, CDs Sue owns)
(16, 35), (16, 34), or (16, 33)

Sue cannot have 32 or fewer because she has more than what Bill has. Bill cannot own more than 16 because of the other limitations. $B + S \leq 51$would describe the total number possible. So these are the only three possibilities.

MATHEMATICAL SYSTEMS AND NUMBER THEORY

5 – 8

Mathematical Systems and Number Theory
What All Students "Should Know"
5 – 8

1. Commutative, associative and distributive properties.

Clarifications:
All students should know:
Commutative property for:

- Addition–
  $a + b = b + a$ or $4 + 6 = 6 + 4$
- Multiplication–
  $a\ b = b\ a$ or $8 \times 3 = 3 \times 8$
- Commutative is to commute for example the distance from Richmond to Kansas City is the same as the distance from Kansas City to Richmond.

Associative property for:

- Addition–
  $(a + b) + c = a + (b + c)$ or $(5 + 10) + 3 = 5 + (10 + 3)$
- Multiplication–
  $(a\ b)\ c = a\ (b\ c)$ or $(4 \times 5)\ 2 = 4\ (5 \times 2)$
- Associative is to associate two numbers together with brackets, etc. An example of the associative property would be if a person is sitting between two people, the person may talk to one person for a while then turn to talk to the other person. They are associating with one another, one at a time.

Distributive property of multiplication over addition:

- $a\ (b + c) = a \cdot b + a \cdot c$ or $6\ (10 + 4) = (6 \times 10) + (6 \times 4)$
- Distributive is to distribute something to each of the parts, for example a businessperson distributed pencils to all the students in the school.

2. Properties of zero and one.

Clarifications:
All students should know:

- Zero product property:
  Any number times 0 equals zero-
  $a \times 0 = 0$
  or
  $8 \times 0 = 0$

- Identity properties:
  Multiplication, any number times 1 equals the number–
  a x 1 = a and 1 x a = a
  or
  16 X 1 = 16 and 1 X 16 = 16
- Addition, any number plus 0 equals the number:
  a + 0 = a and 0 + a = a
  or
  12 + 0 = 12 and 0 + 12 = 12

- Division by zero is undefined:
  for any number divided by zero an answer does not exist–
  for a ÷ 0 or $\frac{a}{0}$ an answer does not exist and is undefined.
  or
  for 9 ÷ 0 or $\frac{9}{0}$ an answer does not exist and is undefined.

Teacher Note:
Students should be able to explore this concept and see how division by zero leads to an answer of infinity, which is not a real solution. This can serve as an approachable underpinning to limits and calculus. One method of exploration would be to start a pattern problem like:

$\frac{10}{10}=1 \qquad \frac{10}{1}=10 \qquad \frac{10}{.1}=100 \qquad \frac{10}{.01}=1000$

Where students can see that the denominator is not only getting smaller, but is approaching zero. As this occurs the answer is going to infinity.

3. Patterns may be used to describe relationships for multiples, factors and exponents.

Clarifications:
All students should know that relationships can be described by:
- Patterns which occur in the units digits–

$4^1 = \underline{4}$	$6^1 = \underline{6}$	$9^1 = \underline{9}$	$2^1 = \underline{2}$
$4^2 = 1\underline{6}$	$6^2 = 3\underline{6}$	$9^2 = 8\underline{1}$	$2^2 = \underline{4}$
$4^3 = 6\underline{4}$	$6^3 = 21\underline{6}$	$9^3 = 72\underline{9}$	$2^3 = \underline{8}$
$4^4 = 25\underline{6}$	$6^4 = 129\underline{6}$	$9^4 = 656\underline{1}$	$2^4 = 1\underline{6}$
Powers of 4 end in 4 or 6.	Powers of 6 end in 6.	Powers of 9 end in 9 or 1.	Powers of 2 are even.

Patterns resulting in an even number and odd number of integer factors–

4 has factors of 1, 2, 4
8 has factors of 1, 2, 4, 8
16 has factors of 1, 2, 4, 8, 16
32 has factors of 1, 2, 4, 8, 16, 32
64 has factors of 1, 2, 4, 8, 16, 32, 64

5 has factors of 1, 5
10 has factors of 1, 2, 5, 10
15 has factors of 1, 3, 5, 15
20 has factors of 1, 2, 4, 5, 20
25 has factors of 1, 5, 25

Conjecture: Perfect squares have an odd number of factors. Justify your answer.

All students should know:

- Conjecture is a statement of what is believed to be happening with the evidence that has been collected or with the pattern at the time the statement is written.
- Counterexample is a situation that does not support the conjecture. It only takes one situation in the evidence or pattern not supporting the conjecture for the conjecture to be determined not to be valid. One example would be the statement $n^2 > n$. When one substitutes in numbers like 5, 6, 10, for n this statement is appears to always be true. However what happens when n = .5 or .25? These would be counterexamples to a statement that appeared to be always true.

4. Order of Operations.

Clarifications:
All students should know the order of operations which consists of operations in the following order:

- Simplify within the grouping symbols (parenthesis, fraction bar).
- Simplify the terms containing exponents.
- Perform multiplication and division operations going from left to right.
- Perform addition and subtraction operations going from left to right.
- Why “Please Excuse My Dear Aunt Sally” is not enough without an understanding of the above. Where the Please stands for parenthesis, the Excuse for exponents, My Dear for multiplication and division, and Aunt Sally stands for addition and subtraction.

This mnemonic device can confuse students into believing that the suggested order must be followed (e.g., multiplication always comes before division) which is not the case.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: A

Evaluate the need for and applications of numbers not contained in the set of whole numbers.

<u>Problem 1</u>:

Process Standards: 3.4, 3.8, and 4.1

You are planning a Sub-Sandwich Party. The quoted cost for 6 people is $12.75 (including tax). Calculate the party cost for sub sandwiches if 15 people, including yourself, are to attend the party.

Solution Notes:

Answer: $31.88 (with rounding after the last step).

Method 1: Per Person Cost $12.75 ÷ 6 = $2.125 $2.125 X 15 = $31.875 = $31.88

$12.75 ÷ 6 = $2.125

round to $2.13 $2.13 X 15 = $ 31.95

Method 2: Proportions

$$\frac{\$12.75}{6} = \frac{X}{15}$$

$$X = \$31.88$$

Method 3: Use # of parts

$$\frac{15}{6} = 2\tfrac{1}{2} \text{ parts}$$

$$2\tfrac{1}{2} * \$12.75 = \$31.88$$

Method 4: Model

6 people	$12.75
6 people	$12.75
3 people	$ 6.38 or $6.375
Total	$31.88

Prerequisites:

All students should:

1. Know how to solve proportions.
2. Know how to add decimals.
3. Be able to do multiplication of decimals and/or fractions.
4. Be able to change fractions to decimals.
5. Be able to round numbers to the nearest cent.
6. Be able to demonstrate an understanding of when to round and the impact of rounding too soon or too often.

<u>Problem 2</u>:

Process Standards: 3.4 and 4.1

At International Falls Minnesota the 6:00 a.m. temperature was -35° F. At 11:00 a.m. the temperature was -19° F. Determine whether the temperature rose or fell and by how many degrees.

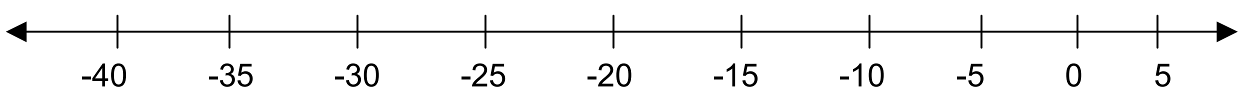
Solution Notes:

Method 1: Adding Integers

$$-35 + X = -19$$
$$X = -19 + 35$$

$X = 16$ the temperature rose 16 ° F.

Method 2: Draw a number line. Plot points by moving from - 35 to -19 yield 16 units to the right.



Method3: Show markings that demonstrate the changes in temperature.

Prerequisites:

All students should know how to:

1. Add and subtract integers.
2. Add and subtract on a number line.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: B
Develop an understanding of and explain order
and relationship among integers, fractions and decimals.

<u>Problem 1</u>:
Process Standards: 1.6 and 1.8

Chris, Ashley, and Alex have just finished practicing basketball. Their shooting statistics are listed below.

Name	Attempts	Number Made	Fraction	Decimal
Chris	12	7		
Alex	8	5		
Ashley	15	9		

Complete the table and list their statistics in order from least to greatest.

Solution Notes:

Name	Attempts	Number Made	Fraction	Decimal
Chris	12	7	$\frac{7}{12}$	0.583
Alex	15	9	$\frac{9}{15}$	0.6
Ashley	8	5	$\frac{5}{8}$	0.625

<u>Order of Shooting Statistics from least to greatest</u>:
Chris - 0.583, Ashley - 0.6, Alex - 0.625

Which representation shows you the order best? Justify your selection.

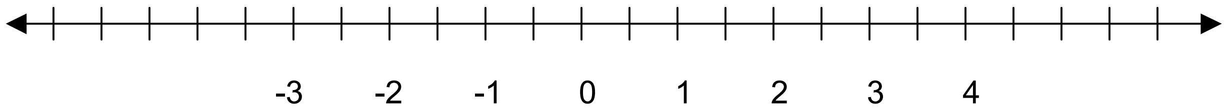
Prerequisites:
All students should know:
1. How to work with ratios and to change fractions to decimals.

<u>Problem 2</u>:

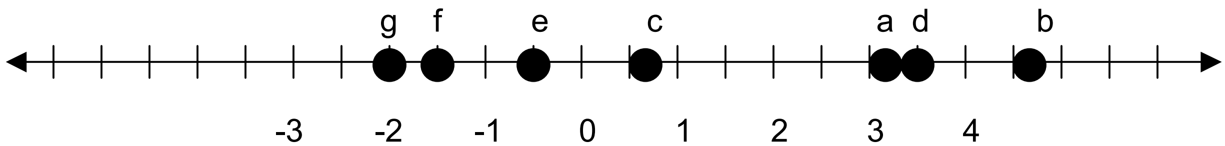
Process Standards: 1.6 and 1.8

Given the number line shown below, place the following numbers in their appropriate position.

$$a.\ \Pi,\ \text{b. } 4.7,\ \text{c. } \frac{7}{11},\ \text{d. } \frac{7}{2},\ \text{e. } -.49,\ \text{f. } -\frac{6}{4},\ \text{g. } -2$$



Solution notes:



Prerequisites:

All students should know:

1. The relative size and placement of real numbers.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: C
Use real-world and mathematical problem situations
to develop and apply number theory concepts
(such as primes, factors, and multiples)

Problem 1:
Process Standards: 1.6 and 1.10

Use a hundreds chart to distinguish prime and composite numbers (Sieve of Eratosthenes).

1. Circle 2; cross out all multiples of 2.
2. Circle 3; cross out all multiples of 3.
3. Circle 5; cross out all multiples of 5.
4. Circle 7; cross out all multiples of 7.

Continue the pattern of numbers until all the numbers that are left are prime.

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
20	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**
30	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
40	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**
50	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**
60	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**
70	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**
80	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**
90	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**

Solution Notes:

Hundreds Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Prime Numbers are the only numbers circled.

Describe eight patterns that this diagram contains.

Are perfect squares considered prime or composite? Explain why or why not.

Students should be able to explain that perfect squares, other than the number 1, by definition must be composite because they will have some number times itself. (One and itself as factors which will make it have more than two factors.)

Prerequisites:
All students should know the definitions of:

1. Prime numbers.
2. Composite numbers.
3. Multiples of numbers.

Problem 2:
Process Standards: 1.6 and 3.3

Mrs. Jones has a class of 25 students. Mr. Smith has a class of 20 students. For an assembly they want to seat their students in an equal number of rows so that Mrs. Jones' sits on one side of an aisle, and Mr. Smith's class sites on the other side of the aisle. Mrs. Jones plans on having the same number of students in each row. Mr. Smith also plans on having the same number of students in each row. Give the maximum number of rows they can use and how many students from each class will be in a row.

Solution Notes:

Method 1:

Factors	25	1 X 25		5 X 5
	20	1 X 20	2 X 10	4 X 5

Greatest Common Factor (GCF) = 5
5 rows with Mrs. Jones having 5 students in each row.
5 rows with Mr. Smith having 4 students in each row.

Method 2:
Draw Model

Left Side of the Aisle	Aisle	Right Side of the Aisle
X X X X X		X X X X
X X X X X		X X X X
X X X X X		X X X X
X X X X X		X X X X
X X X X X		X X X X

Mrs. Jones' class sits in 5 rows with 5 students in a row.

Mr. Smith's class sits in 5 rows with 4 students in a row.

Problem 3:
Process Standards: 1.6 and 3.3

The eighth grade track team decided to do a jog-a-thon where the team would take turns jogging (each member jogging) for 30 minutes. To make things interesting, the team chose to have teammates run in pairs. One pair was Sheila, a speedster, and Samson, the shot-putter. Sheila could jog at a 6-minute mile pace while Samson ran at an 8-minute mile pace. (They will be running on the school's quarter-mile track.)

How many times during their run will Sheila and Samson both pass the starting point at the same time? Justify your solution.

Solution notes:
They will both be at the starting point 5 times during their jog. Student should be able to justify their solutions by either graphing or computing the number of times each jogger made laps and at what time. Sheila would pass the starting line at 1.5, 3, 4.5, 6, … minutes while Samson passed at 2, 4, 6, …minute intervals. They should both be at the starting point at 6, 12, 18, 24, and 30 minutes.

Prerequisites:
All students should know:

1. The definition of factors of numbers.
2. The definition of multiples of numbers.
3. How to use factors and multiples to solve real-world problems.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: D

Realize the dynamic nature of mathematics and how different mathematical systems apply to current and developing real-world situations.

<u>Problem 1</u>:

Process Standards: 1.2, 1.10, 2.1, and 4.8

Choose a topic and explain how mathematics impacts real world situations. Suggested topics include:

- Stock market
- Currency
- Binary Systems
- Population Growth
- Careers
- Patient Dosages
- Fiber Optics
- Space Explorations
- Environmental Issues
- Taxes

Solution Notes:
The answers will vary from student to student.

<u>Problem 2</u>:

Process Standards: 1.2, 1.10, 2.1, and 4.8

Choose a stock from the newspaper or computer. Create a graph displaying its changes over a 5-day period. (This activity could be extended to as long a period of time as you feel comfortable spending on Stock Market.)

Solution Notes:
Any graph which accurately depicts the data collected.

Prerequisites:
All students should:

1. Know how to add and subtract fractions.
2. Be able to create graphs to represent a real-world situation.

Problem 3:

Process Standards: 1.2, 1.10, 2.1, and 4.8

Given that the following problem is a true statement, which base would the numbers have to be written in for the given solution to be correct?

$$\begin{array}{r} 3465 \\ +\ 2503 \\ \hline 6301 \end{array}$$

Solution notes:
The student should be able to find that this is base 7. The carrying in the second column from the right should provide the greatest clue.

Prerequisites:
All students should:

1. Know how to calculate with bases other than 10.
2. Know the impact bases have on the solution of problems.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: E

Apply commutative, associative, and distributive relationships in computation and estimation situations.

<u>Problem 1</u>:
Process Standards: 1.6, 3.3, and 3.6

Investigate the commutative property for real numbers and determine if it applies to the four operations of addition, subtraction, multiplication, and division. Give five examples of each operation, unless a counterexample is found.

Solution Notes:
Accept any solution, which concludes that the commutative property is true for addition and multiplication, but false for subtraction and division.

<u>Problem 2</u>:
Process Standards: 1.6 and 3.3

Show two different ways the distributive property can be used to determine the bottom surface area of a rectangular 12-inch by 16-inch cake pan.

Solution Notes:

$$12 \times 16 = 12\,(10 + 6) = 12\,(10) + 12\,(6)$$
$$120 \quad + 72 \quad = 192 \text{ square inches}$$

$$16 \times 12 = 16\,(10 + 2) = 16\,(10) + 16\,(2)$$
$$160 \quad + 32 \quad = 192 \text{ square inches}$$

<u>Problem 3</u>:
Process Standards: 1.6 and 3.3

Create a new operation a @ b (where a @ b = 2a – 3b) and determine whether or not the commutative, associative, and the distributive properties work for this operation.

Solution Notes:
Answers may vary, but students should justify that these properties will not hold for this operation where a and b are not equal. They should provide examples to validate this conclusion.

Prerequisites:
All students should:

1. Know and be able to apply the field properties.

Mathematical Systems and Number Theory
What All Students "Should Do"
5 – 8

Written Benchmark: F

Recognize the connection of irrational numbers and the real world.

<u>Problem 1</u>:

Process Standards: 1.6, 2.3, 3.5, 3.6, and 4.6

Work with a partner. Investigate the relationship of circumference to diameter to discover Pi (Π). Measure the circumference and diameter of 6 circles. Construct a table, which includes the measurements, the ratio of the circumference to the diameter, and the decimal equivalent. Round the decimal to a place value of two digits to the right of the decimal point. Compare the results. Discuss your results with other members of the class.

Solution Notes:

All decimal answers should be near Pi (π).

Object	Circumference	Diameter	C / d	Decimal

Prerequisites:

All students should:

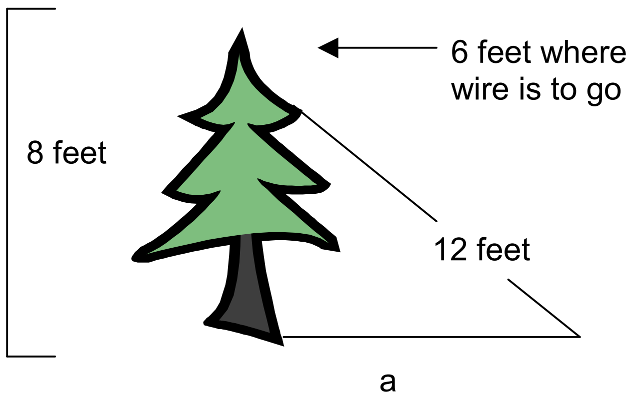
1. Define Pi as an irrational number.
2. Know the definitions of circumference and diameter.
3. Know measurement techniques.

<u>Problem 2</u>:
Process Standards: 1.5, 1.6, 1.8, and 3.2

Use the Pythagorean theorem to solve the following problem**:**

A tree on your property is beginning to grow crooked. You plan to attach a wire 2 feet (at the 6 feet mark) from the top of the tree to straighten the tree. The tree is 8 feet tall. The wire is 12 feet long. How far from the base of the tree must you stake the wire? Illustrate the problem.

Solution Notes:



$a^2 + b^2 = c^2$

$a^2 + 6^2 = 12^2$

$a^2 + 36 = 144$

$a^2 = 108$

$a = \sqrt{108}$

$a = 10.39$ ft.

Prerequisites:
All students should:
1. Know how to use the Pythagorean theorem.

DISCRETE MATHEMATICS

5 – 8

Discrete Mathematics

Discrete Mathematics can be more easily interpreted by thinking in five categories:

- Networks and Pathways
- Counting - combinations / permutations
- Fair Division
- Logic and Reasoning
- Matrices
-

The clarifications for the knowledge statements are related to these five categories.

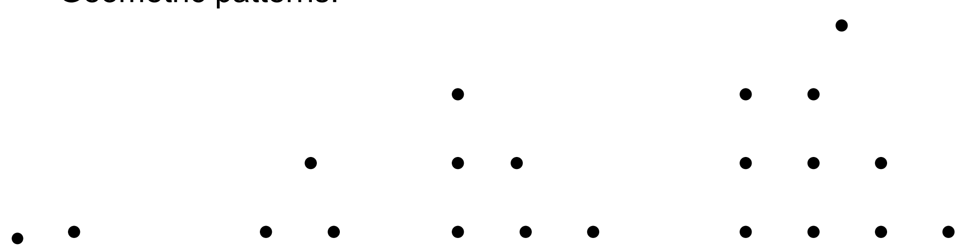
Discrete Mathematics
What All Students “Should Know”
5 – 8

1. Definition and examples of patterns.

Clarifications:

All students should know example of:

- Geometric patterns:



- Numeric patterns:

  5, 8, 11, 14 **. . .**

  1, 4, 9, 16, 25, 36, 49 **. . .**

  - 2, - 4, - 6, - 8, - 10 **. . .**

  - 10, - 5, 0, 5, 10, 15 **. . .**

  1, 3, 6, 10 **. . .**

- Analogy:

  Circle is to circumference as triangle is to perimeter.

  Polygon is to the number of sides ***n*** as pentagon is to 5 sides.

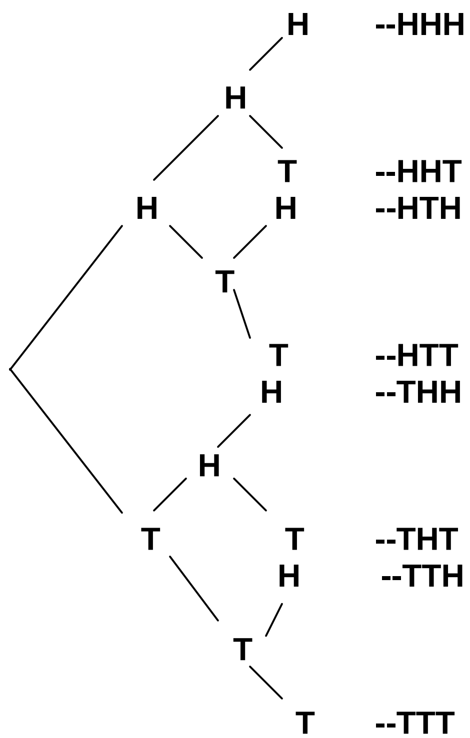
2. Definition and examples of tree diagrams.

Clarifications:

All students should know:

- Tree diagrams are organizational tools to keep track of counting techniques, family trees, etc.
- An example of a tree diagram:

Outcomes of a Three-Coin Toss

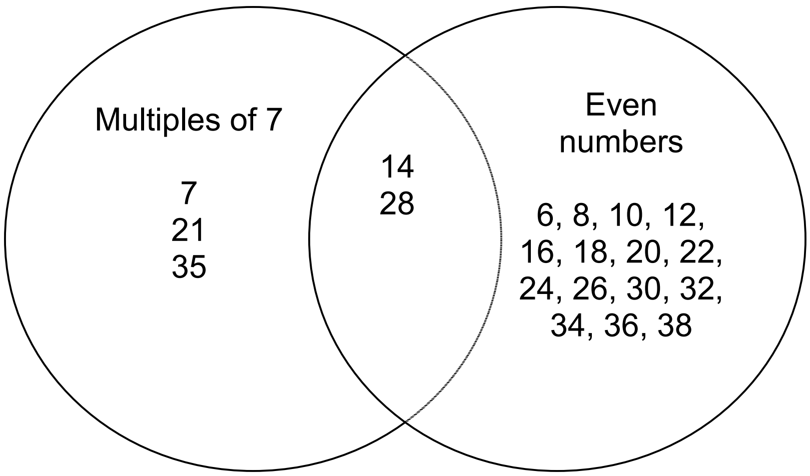


3. Definition and examples of Venn Diagrams.

Clarifications:
All students should know:

- A Venn diagram is a mathematical model used to show set relationships.
- Examples of a Venn Diagram to show a relationship.

Show the relationship of multiples of 7 up to 35 to the set of even numbers {6 - 38}.

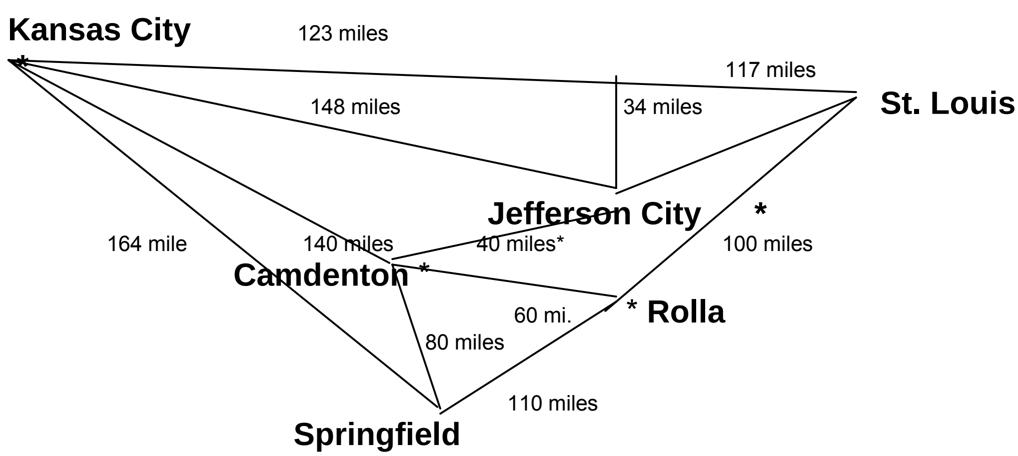


4. Definition and examples of networks.

Clarifications:
All students should know:

- Networks are mathematical models that show all possible outcomes.
- Examples of Networks include:
  - ♦ A telephone tree
  - ♦ A family tree
  - ♦ A tree diagram to count combinations
  - ♦ Travel routes

Find the shortest route possible to visit all of the cities shown on the diagram.



- ♦ Tournament brackets
- ♦ Sports schedule
- ♦ Matrices

Discrete Mathematics
What All Students "Should Do"
5 – 8

Written Benchmark: A
Determine and continue a pattern using inductive reasoning.

<u>Problem 1</u>:
Process Standards: 1.6 and 3.5

Sketch the next figure in the pattern in box provided for Figure 4:

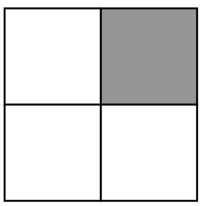


Figure 1

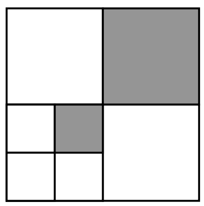


Figure 2

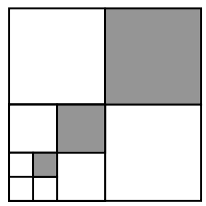


Figure 3

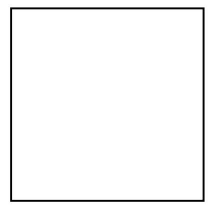


Figure 4

Solution Notes:

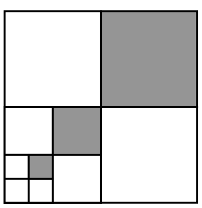


Figure 4   (The smallest square has one-fourth shaded.)

Prerequisites:
All students should:

1. Know how to use logical reasoning and critical thinking strategies.

<u>Problem 2</u>:
Process Standards: 1.6 and 3.5

Identify the next four numbers in the pattern. In the box below, explain how you determined the next four numbers in the pattern:

3,   9,   11,   33,   35,   ___,   ___,   ___,   ___

Solution Notes:
The next four numbers in the pattern are: 105, 107, 321, 323. To get the sixth term you multiply 35 (the 5^{th} number) by three, and to get the seventh term you add on two to the sixth number in the pattern, continue to multiply by three then add on two for every other term. This is the same pattern that started with the first three terms.

Other solutions are possible. Teachers should check student justifications for logical reasoning and strong use of mathematical communication.

Prerequisites:
All students should:
1. Be able to use basic computation skills.
2. Be able to reason logically.
3. Know how to use critical thinking.
4. Be able to work with patterns.

Discrete Mathematics
What All Students "Should Do"
5 – 8

Written Benchmark: B

Look at if-then relationships to make logical deductions.

<u>Problem 1</u>:

Process Standards: 1.5 and 3.3

Judy, Stephanie, Steve, and Michael are four middle school students. Each has a different favorite class. One likes Math, one likes Physical Education, one likes French, and one likes Band. Michael cannot read a note of Music. Steve is a starter on the Basketball Team. Judy made an A on yesterday's Spanish Test. Stephanie will tour Paris with the French Club during summer vacation. Identify each person's favorite subject.

Solution Notes:

	Math	Physical Education	French	Band
Judy		3	4	
Stephanie		3		
Steve	3	2	3	3
Michael		3		1

1. This box is "no" from sentence 1, "Michael cannot read a note of music"
2. This box is "yes" because of sentence 2.
3. These boxes are "no" because of number 2.
4. This box is "no" because of sentence 3.

Continue in the same manner to complete the problem.

Judy - Band
Stephanie - French
Steve - Physical Education
Michael - Math

Prerequisites:

All students should:

1. Be able to use the process skills of problem solving and logical reasoning.

<u>Problem 2</u>:
Process Standards: 1.6 and 3.6

The only person to get on the school bus at the first stop was Zeno. He noticed that two people got on at the second stop, four people got on at the third stop, and eight people got on at the fourth stop. If the pattern continues, and the bus can hold 65 people, how many stops can the bus Zeno rides make before it runs out of available seats? Show how you arrived at the answer in the space provided below.

Solution Notes:
1 + 2 + 4 + 8 + 16 + 32 = 63; pictures are acceptable.
$2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 63$ (Optional Answer)

The bus will make 6 stops to be filled with 63 students. If the bus makes the 7^{th} stop 64 would get on and the bus only had room for two more students.

Prerequisites:
All students should:
1. Be able to use the process skills of problem solving, communication, reasoning, and connections.
2. Know how to think logically.
3. Be able to use critical thinking skills.

Discrete Mathematics
What All Students "Should Do"
5 – 8

Written Benchmark: C

Investigate tree, Venn, or student-developed diagrams
as an organizing tool for problem solving.

<u>Problem 1</u>:
Process Standards: 1.6, 1.8, and 3.2

The Sub Shop has a special price on Sub Sandwiches using one meat, one cheese, and one topping on each sandwich. Construct a diagram to find the number of different sandwiches you could purchase for the special price.

Choices:
Meat: turkey, ham, roast beef
Cheeses: American cheese, Swiss Cheese
Toppings: lettuce, tomato, onion

Solution Notes: 18 sandwiches

```
                                                  ------------Onion
          American Cheese------------------------ ------------Tomato
                                                  ------------Lettuce
Turkey----------------------------
                                                  ------------Onion
          Swiss Cheese--------------------------- ------------Tomato
                                                  ------------Lettuce

                                                  ------------Onion
          American Cheese------------------------ ------------Tomato
                                                  ------------Lettuce
Ham-------------------------------
                                                  ------------Onion
          Swiss Cheese--------------------------- ------------Tomato
                                                  ------------Lettuce

                                                  ------------Onion
          American Cheese------------------------ ------------Tomato
                                                  ------------Lettuce
Beef------------------------------
                                                  ------------Onion
          Swiss Cheese--------------------------- ------------Tomato
                                                  ------------Lettuce
```

Prerequisites:
All students should:
1. Know how to use tree diagrams as a problem-solving tool.

<u>Problem 2</u>:

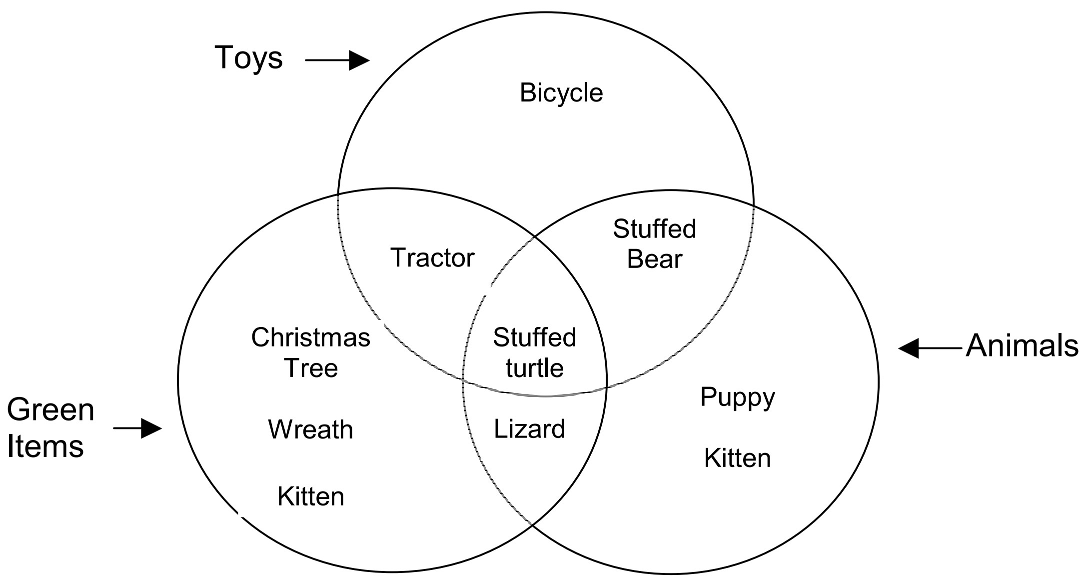
Process Standards: 1.5, 1.8, 2.1, and 3.5

Use a diagram to find the similarity and differences of real world products found in a catalog. [Teacher Notes: At the winter holiday season; select three attributes like toys, animals, and green items for students to find in catalogs. Students could cut out the items from a catalog and paste them on poster board, which has a 3-circle diagram.] Show as a model, since a Venn diagram is not the only possible choice, then save student diagrams for samples for future year's models.

Solution Notes:

EXAMPLE OF A VENN DIAGRAM

Use pictures not words.



Solution:

<u>Similarities</u> — The stuffed bear is both a toy and an animal. The tractor is both a toy and a green item; etc.
<u>Differences</u> — The Christmas tree is green but not a toy or an animal. The wreath is green but not a toy or an animal. The lizard is green and is an animal, but not a toy; etc.

Prerequisites:

All students should know how to:

1. Use a diagram as a problem-solving tool.

Problem 3:
Process Standard: 3.8

The Student Council and Math Clubs are responsible for two activities a year, a School Dance and a Math Contest. The committees have budgeted the following amounts:

Math Committee		
Item	Dance	Contest
Food	\$75.00	\$150.00
Beverage	\$20.00	\$45.00
Decorations	\$40.00	\$125.00

Student Council		
Item	Dance	Contest
Food	\$30.00	\$60.00
Beverage	\$15.00	\$20.00
Decorations	\$10.00	\$55.00

Create or complete the matrix, which includes the total budget costs for both committees.

Item	Dance	Contest
Food		
Beverage		
Decorations		
Totals		

Solution Notes:
Total Budgeted Costs

Item	Dance	Contest
Food	\$ 105.00	\$ 210.00
Beverage	\$ 35.00	\$ 65.00
Decorations	\$ 50.00	\$ 180.00
Totals	\$ 190.00	\$ 455.00

Prerequisites:
All students should be able:

1. To create a matrix and to add matrices.

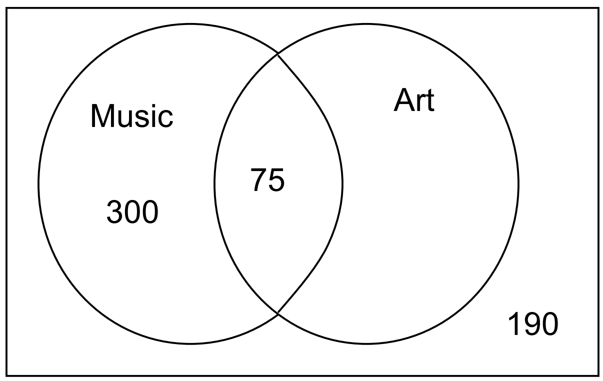
Problem 4:

Process Standard: 3.8

A middle school has 748 students. 300 of them are enrolled in music, but not art and 75 are enrolled in both art and music. 190 students are not enrolled in either music or art. How many are enrolled in art, but not music?

Solution notes:

183 are enrolled in music and not art. This problem could be solved using a Venn diagram.



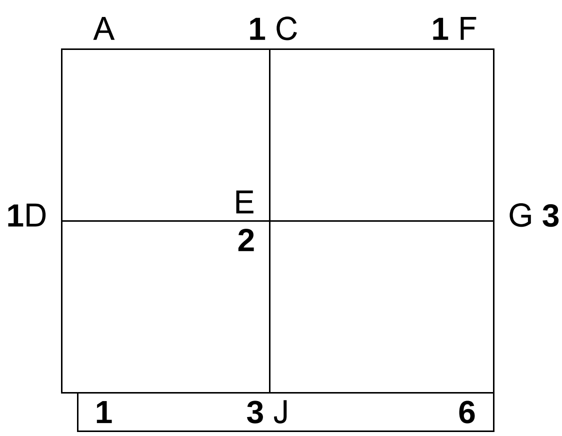
Discrete Mathematics
What All Students "Should Do"
5 – 8

Written Benchmark: D
Explore transportation networks.

Problem 1:
Process Standards: 1.5, 1.10, and 3.4

Use Pascal's Triangle to discover the relationships between the triangle and discovering the number of paths in a rectangular or square pathway. First work with Pascal's Triangle. Give students the first four lines and have them develop the next three lines. Use the triangle to solve the following problems. Find the number of ways you can get from Point A to Point B. [Teacher note: you could also give this problem without the "Use Pascal's Triangle" as a prompt.]

Solution Notes:
The edges representing only one path can get you from A to C and A to D. However, as we get from A to E, we are able to follow 2 paths (A, D, E, or A, C, E). To go from A to G, we are able to follow 3 paths (A, D, E, G; A, C, E, G; A, C, F, G)



This will work for any square or rectangle. This variation of Pascal's Triangle is a rotation of the normal display of this pattern. The pathway described above is shown below using a dotted line as the various paths.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Prerequisites:
All students should:

1. Have had experience working with Pascal's Triangle.

<u>Problem 2</u>:

Process Standards: 1.6, 2.1, 3.2, and 3.6

Locate a city that is at least 100 miles away from your school. Using a map, choose 3 different routes. Rank the routes from shortest to longest. Find the distance from, your school to the city using each route. Show each route as a network.

Solution Notes:
The answers will vary since the students may select different cities. The teacher should create or select a map for this activity. [Teacher note: check with Social Studies teachers. Perhaps this could be connected with their unit of study].

Prerequisites:
All students should:

1. Be able to use the scale on a map.
2. Be able to read the measurements on a map.
2. Have the map skills necessary to use a map successfully.